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# ON CONTINUOUS IMAGES OF DENSE SUBSETS OF $\Sigma$-PRODUCTS

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Abstract. In this paper we consider regular images of dense subsets of $\Sigma$-products with "geometric" properties, and with these we obtain a similar result to a classical theorem of M.G. Tkachenko.

## 1. Introduction

Since a continuous map can distort its domain, it is natural to ask which properties are preserved. The answer to this question depends to a great extent on the nature of the map's domain and range.

H. H. Corson, in [3] was the first to investigate properties of $\Sigma$-products, which are dense subspaces of topological products. Some of his open questions were about properties of $\Sigma$-products of copies of the rational numbers and their continuous images. Based in his work, researchers began to consider continuous images of $\Sigma$-products, since their "geometric" properties allowed to obtain and improve some results, instead of considering products of spaces as the domain of maps.

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This area was systematically investigated by B. A. Efimov, A. Kombarov, R. Engelking and others. An important result was obtained by B.A. Efimov in 1963, who proved that a compact space which is a continuous image of a $\Sigma$-product of a family of compact metrizable spaces is itself metrizable. Nevertheless, his techniques could not be used to solve some problems on continuous images of dense subspaces of $\Sigma$-products of metrizable spaces.

A new result in this context was obtained by M.G. Tkachenko in [10], who proved, among other things, that under the hypothesis $2^\omega < 2^{\omega_1}$, compact spaces which are continuous images of dense subsets of $\Sigma$-products of spaces with countable net weight are metrizable. Finally in 1982 M.G. Tkachenko in [11] proved the following result without extra axioms.

**Theorem 1.1.** *(Tkachenko) Let $X = \prod_{i\in I} X_i$ be a product of spaces such that $nw(X_i) \leq \omega$ for every $i \in I$, $p \in X$, $S \subseteq \Sigma(p)$ a dense subset of $\Sigma(p)$ and $f : S \to Y$ a continuous map onto a Tikhonov space $Y$ of pointwise countable type. Then $nw(Y) \leq \omega$. In particular, if $Y$ is compact, then $Y$ is metrizable.*

Theorem 1.1 is one of the strongest and deepest results in this area. Others works by M.G. Tkachenko (see [10], [11] and [12]) contain more results on the theory of continuous images of dense subsets of products spaces. In the proof of Theorem 1.1, the assumption about the image space $Y$ being a Tikhonov space plays an important role. In spite of this, some of the techniques employed have been used here to get the results in Section 4. The main aim of this work is to weaken the condition imposed on the image $Y$ in Theorem 1.1 substituting complete regularity by regularity. The main results in this paper show that Tkachenko's theorem is still valid if the space $Y$ is assumed to be only regular, but considering special dense subsets of $\Sigma$-products.

## 2. Notation and terminology

We define here some of the concepts which will be used later, the concepts not defined here are well known. In any case the reader can consult [7]. All space are assumed to be Hausdorff.

The symbol $\mathbb{I}$ denotes the unit closed interval $[0, 1]$ with the usual topology and $\mathbb{N}$ denotes the natural numbers.

If $X$ is a topological space and $A \subseteq X$, $\tau(A,X)$ denotes the collection of all open subsets of $X$ which contain the set $A$. If $A = \{x\}$, we write $\tau(x,X)$ instead of $\tau(\{x\},X)$. A set $Y \subseteq X$ is called *$G_\delta$-dense* in $X$ if $Y$ intersects every non empty $G_\delta$-set of $X$.

Let $X = \prod_{i\in I} X_i$ be a product of spaces. Given a non-empty set $J \subseteq I$, we define the function

$$\pi_J : \prod_{i\in I} X_i \to X_J = \prod_{i\in J} X_i$$

which is the natural projection. If $Y \subseteq X$, then $Y_J = \pi_J(Y)$. We will write $Y_i = \pi_i(Y)$ if $J = \{i\}$ for some $i \in I$.

Let $p \in \prod_{i\in I} X_i$ be a fixed point. We define the *support* of a point $x \in X$ in the following way:

$$\mathrm{supp}(x) = \{i \in I : \pi_i(x) \neq \pi_i(p)\}.$$

If $Y \subseteq X$ we define $\mathrm{supp}(Y) = \bigcup_{x\in Y} \mathrm{supp}(x)$.

The $\Sigma$-product $\Sigma(p)$ of a family of spaces $\{X_i : i \in I\}$ is the following subspace of $\prod_{i\in I} X_i$

$$\Sigma(p) = \{x \in X : |\,\mathrm{supp}(x)| \leq \omega\}.$$

Given $S \subseteq \Sigma(p)$ and $J \subseteq I$, we define the following subsets of $S$:

$$\underline{S}[J] = \{x \in S : \mathrm{supp}(x) \subseteq J\} \text{ and}$$
$$\overline{S}[J] = \{x \in S : \mathrm{supp}(x) \cap J = \emptyset\}.$$

## 3. Auxiliary results

We introduce a class of subsets of $\Sigma$-products. Part (1) of the following definition was introduced by Engelking (see [5, Section 3] and [8]).

**Definition 3.1.** Let $X = \prod_{i\in I} X_i$ be a product of spaces and $p \in X$ be a fixed point. Consider $x, y \in X$, $x \neq y$ and $J \subseteq I$ countable. We define the point $z(J,x,y) \in X$ as follows:

$$\pi_i(z(J,x,y)) = \begin{cases} \pi_i(x) & \text{if } i \in J; \\ \pi_i(y) & \text{if } i \notin J. \end{cases}$$

(1) We call a set $S \subseteq X$ *invariant* if for every pair of points $x, y \in S$ and every countable $J \subseteq I$, the point $z(J,x,y)$ belongs to $S$.

(2) We call a set $S \subseteq X$ *almost invariant with respect to* $p$ if for every $x \in S$ and every countable $J \subseteq I$, the point $z(J, x, p)$ belongs to $S$.

It is easy to see that if a set $S$ is invariant, then it is almost invariant with respect to any point $p \in S$. The next example shows that there exist dense almost invariant dense sets which are not invariant.

**Example 3.2.** Let us consider the set $\Sigma(\overline{0}) \subseteq \mathbb{I}^{\mathfrak{c}}$, where $\overline{0}$ is the point of $\mathbb{I}^{\mathfrak{c}}$ all whose coordinates are zero. We fix $\alpha_0$ and $\alpha_1$, two different elements of $\mathfrak{c}$. Let $Y = \{x \in \Sigma(\overline{0}) : \pi_{\alpha_0}(x) = \pi_{\alpha_1}(x) = 1\}$ and $S = \Sigma(\overline{0}) \setminus Y$.

It is easy to verify that the set $S$ is almost invariant with respect to $\overline{0}$ and is dense in $\Sigma(\overline{0})$. Let $J = \{\alpha_0\}$. We define points $x_0, x_1 \in S$ as

$$\pi_\alpha(x_i) = \begin{cases} 1 & \text{if } \alpha = \alpha_i; \\ 0 & \text{otherwise.} \end{cases}$$

Note that there is no point $s \in S$ such that $\pi_J(s) = \pi_J(x_0)$ and $\pi_{\mathfrak{c} \setminus J}(s) = \pi_{\mathfrak{c} \setminus J}(x_1)$. Therefore the set $S$ is not invariant.

The next two lemmas are stated without proof, since they are easily verified or can be found in the existing literature.

**Lemma 3.3.** *Let $X$ be a regular topological space and $Y \subseteq X$ a Lindelöf subspace of $X$. For every point $x \in X \setminus Y$, there exists a closed set $P$ of type $G_\delta$ in $X$ such that $x \in P \subseteq X \setminus Y$.*

**Lemma 3.4.** *Let $X$ be a topological space and $Y \subseteq X$. If $Y$ is a Lindelöf $G_\delta$-dense subspace of $X$, then $X = Y$.*

**Lemma 3.5.** *Let $X = \prod_{i \in I} X_i$ be a product of separable spaces, $S$ dense in $X$ and $U$ open in $S$. Then there exists a countable set $J \subseteq I$ such that* $\mathrm{cl}_S\, U = S \cap \pi_J^{-1}(\pi_J(\mathrm{cl}_S\, U))$.

*Proof.* There exists an open set $V$ in $X$ such that $U = S \cap V$. Apply [7, Exercise 2.7.12 a)] to find a countable set $J \subseteq I$ such that $\mathrm{cl}_X\, V = \pi_J^{-1}(\pi_J(\mathrm{cl}_X\, V))$. Since $S$ is dense in $X$, we have that $\mathrm{cl}_S\, U = S \cap \pi_J^{-1}(\pi_J(\mathrm{cl}_S\, U))$. □

The following theorem of A.V. Arhangel'skiĭ, is a well known result and very important in the theory of continuous images of dense subsets of products. We do not state it in its most general form, the reader can find in [1] and [2] more general statements of this result.

**Theorem 3.6.** *(Arhangel'skiĭ) Let $X = \prod_{i\in I} X_i$ be a product of spaces such that $nw(X_i) \leq \omega$ for every $i \in I$, $S$ a dense subset of $X$ and $f : S \to Y$ a continuous map onto a regular space $Y$. Then the set $Q = \{y \in Y : \chi(y, Y) \leq \omega\}$ satisfies $nw(Q) \leq \omega$.*

The following two propositions are easily verified, but state important properties of subsets of Σ-products.

**Proposition 3.7.** Let $X = \prod_{i\in I} X_i$ a product of spaces, $p \in X$, $S \subseteq \Sigma(p)$ and $J \subseteq I$. Assume that a set $T \subseteq S$ satisfies $\mathrm{supp}(T) \subseteq J$. Then the following conditions hold:

(1) $\mathrm{cl}_S\, T \subseteq S[J]$;
(2) the map $\pi_J|_T : T \to \pi_J(T)$ is a homeomorphism.

**Proposition 3.8.** Let $X = \prod_{i\in I} X_i$ be a product of spaces and $p \in X$. A set $S \subseteq \Sigma(p)$ is almost invariant with respect to $p$ if and only if for every countable set $J \subseteq I$, $\pi_J(S[J]) = \pi_J(S)$.

*Proof.* Let $J$ be a non-empty countable subset of $I$. For every $x \in S$ consider the point $z(J, x, p)$ defined in 3.1. Observe that $\pi_J(z(J, x, p)) = \pi_J(x)$ and that $\mathrm{supp}(z(J, x, p)) \subseteq J$. It is easy to see that $z(J, x, p) \in S[J] \subseteq S$ for every $x \in S$ if and only if $\pi_J(S[J]) = \pi_J(S)$. □

The following example shows that Proposition 3.8 cannot be extended to arbitrary dense subspaces of Σ-products.

**Example 3.9.** Consider the point $\overline{0} \in \mathbb{I}^{\mathfrak{c}}$ all whose coordinates are zero. Since the set $\mathcal{A} = \{J \subseteq \mathfrak{c} : |J| \leq \omega, J \neq \emptyset\}$ has cardinality $\mathfrak{c}$, we can write $\mathcal{A} = \{J_\alpha : \alpha < \mathfrak{c}\}$.

Let $\alpha < \mathfrak{c}$. Note that $|\mathbb{I}^{J_\alpha}| = \mathfrak{c}$, and so we can enumerate $\mathbb{I}^{J_\alpha} = \{x^\alpha_\beta : \beta < \mathfrak{c}\}$. We consider also an enumeration of the set $\mathfrak{c} \setminus J_\alpha = \{i^\alpha_\beta : \beta < \mathfrak{c}\}$. For each $\alpha, \beta < \mathfrak{c}$, consider the point $s^\alpha_\beta \in \mathbb{I}^{\mathfrak{c}}$ defined as follows:

$$\pi_i(s^\alpha_\beta) = \begin{cases} \pi_i(x^\alpha_\beta) & \text{if } i \in J_\alpha; \\ 1 & \text{if } i = i^\alpha_\beta; \\ 0 & \text{otherwise.} \end{cases}$$

Let $S = \{s^\alpha_\beta : \alpha, \beta < \mathfrak{c}\}$. Note that $S \subseteq \Sigma(\overline{0})$ and that for every non-empty countable set $J \subseteq I$ we have that $\pi_J(S) = \mathbb{I}^J$.

Let $J \in \mathcal{A}$. Take $x \in \mathbb{I}^J$ such that $\pi_i(x) \neq 0$ or $1$ for every $i \in J$. Given the construction of the set $S$, for every point $s \in S$ there exists at least one $i \in \operatorname{supp}(s)$ such that $\pi_i(s) = 1$. Hence $x \notin \pi_J(S[J])$. We conclude that $\pi_J(S[J]) \neq S_J = \mathbb{I}^J$ for any $J \in \mathcal{A}$. Proposition 3.8 implies that the set $S$ is not almost invariant.

## 4. Main results

With the help of results in Section 3, we are now in the position to prove our main theorem.

**Theorem 4.1.** *Let $X = \prod_{i \in I} X_i$ be a product of spaces such that $nw(X_i) \leq \omega$ for every $i \in I$, $p \in X$, $S \subseteq \Sigma(p)$ be a dense subset and almost invariant with respect to $p$. Let $f : S \to Y$ be a continuous mapping onto a regular space $Y$. Then for every non-empty compact subspace $K \subseteq Y$, there exists a point $y \in K$ such that $\chi(y, K) \leq \omega$.*

*Proof.* We can assume without loss of generality, that $|I| > \omega$ and $|X_i| > 1$ for every $i \in I$. Suppose there is a non-empty compact subspace $K \subseteq Y$ such that $\chi(y, K) > \omega$ for every $y \in K$. In particular, $K$ does not have countable weight. Let $L_0 = \emptyset$. Take a point $y_0 \in K$ such that $f(p) \neq y_0$. Let $F_0$ be a set of type $G_\delta$ in $Y$ such that $y_0 \in F_0$ and $f(p) \notin F_0$. Since $Y$ is a regular space, we can assume, without loss of generality that $F_0 = \bigcap_{n=1}^\infty U_n$, where $U_n \in \tau(y_0, Y)$ and $\operatorname{cl}_Y U_{n+1} \subseteq U_n$ for every $n \in \mathbb{N}$. Take $E_0 = f^{-1}(F_0)$ and $V_n = f^{-1}(U_n)$ for every $n \in \mathbb{N}$. Then we have that $E_0 = \bigcap_{n=1}^\infty V_n = \bigcap_{n=1}^\infty \operatorname{cl}_S V_n$. Lemma 3.5 implies that for every $n \in \mathbb{N}$ there exists a countable set $M_n \subseteq I$ such that $\operatorname{cl}_S V_n = S \cap \pi_{M_n}^{-1}(\pi_{M_n}(\operatorname{cl}_S V_n))$. It is clear that the set $J_0 = \bigcup_{n=1}^\infty M_n$ is countable and the equality $\operatorname{cl}_S V_n = S \cap \pi_{J_0}^{-1}(\pi_{J_0}(\operatorname{cl}_S V_n))$ holds for every $n \in \mathbb{N}$. Therefore

$$E_0 = S \cap \pi_{J_0}^{-1}(\pi_{J_0}(E_0)).$$

Observe that $p \notin E_0$. Suppose there is $x \in E_0$ such that $\operatorname{supp}(x) \cap J_0 = \emptyset$. Then $p \in S \cap \pi_{J_0}^{-1}(\pi_{J_0}(x)) \subseteq S \cap \pi_{J_0}^{-1}(\pi_{J_0}(E_0)) = E_0$. This contradiction implies that $E_0 \cap \overline{S}[J_0] = \emptyset$. Since $K$ is compact, the inequality $\chi(y, K) \leq \chi(y, F_0 \cap K) \cdot \chi(F_0 \cap K, K)$ holds for every $y \in F_0 \cap K$. This inequality and the choice of $K$ imply that $\chi(y, F_0 \cap K) > \omega$ for every $y \in F_0 \cap K$.

Suppose we have defined, for some $\gamma < \omega_1$, nested and increasing sequences $\{L_\alpha : \alpha < \gamma\}$ and $\{J_\alpha : \alpha < \gamma\}$ of countable subsets of $I$, and sequences $\{F_\alpha : \alpha < \gamma\}$, $\{E_\alpha : \alpha < \gamma\}$ of closed sets of type $G_\delta$ in $Y$ and $S$ respectively, such that the following conditions hold for every $\alpha < \gamma$:

*i*) $F_\alpha \cap K \neq \emptyset$ and $\chi(y, F_\alpha \cap K) > \omega$ for every $y \in F_\alpha \cap K$;
*ii*) $F_\alpha \cap f(S[L_\alpha]) = \emptyset$;
*iii*) $E_\alpha = f^{-1}(F_\alpha) = S \cap \pi^{-1}_{J_\alpha}(\pi_{J_\alpha}(E_\alpha))$;
*iv*) $L_\alpha = \bigcup_{\beta<\alpha} J_\beta$ is a proper subset of $J_\alpha$;
*v*) $E_\alpha \cap \overline{S}[J_\alpha \setminus L_\alpha] = \emptyset$.

Let $F(\gamma) = \bigcap_{\alpha<\gamma} F_\alpha$. Note that $F(\gamma)$ is a closed set of type $G_\delta$ in $Y$. Since $K$ is compact, *i*) implies that $F(\gamma) \cap K \neq \emptyset$. Note that $\chi(y, K) \leq \chi(y, F(\gamma) \cap K) \cdot \chi(F(\gamma) \cap K, K)$ holds for every $y \in F(\gamma) \cap K$. This inequality and the choice of the set $K$ imply that $\chi(y, F(\gamma) \cap K) > \omega$ for every $y \in F(\gamma) \cap K$.

Let $L_\gamma = \bigcup_{\alpha<\gamma} J_\alpha$, then $|L_\gamma| \leq \omega$ and the set $Z_\gamma = S[L_\gamma]$ satisfies $nw(Z_\gamma) \leq \omega$. Hence $(F(\gamma) \cap K) \setminus f(Z_\gamma) \neq \emptyset$. Take a point $y_\gamma \in (F(\gamma) \cap K) \setminus f(Z_\gamma)$. By Lemma 3.3 there exists a set $F$ of type $G_\delta$ in $Y$ such that $y_\gamma \in F$ and $F \cap f(Z_\gamma) = \emptyset$. Let $F_\gamma = F \cap F(\gamma)$. Then $F_\gamma$ is also a set of type $G_\delta$ in Y, so we can assume that $F_\gamma = \bigcap_{n=1}^\infty W_n$, where $W_n \in \tau(y_\gamma, Y)$ and $\mathrm{cl}_Y W_{n+1} \subseteq W_n$ for every $n \in \mathbb{N}$. Take $E_\gamma = f^{-1}(F_\gamma)$ and $O_n = f^{-1}(W_n)$ for every $n \in \mathbb{N}$. Then we have that $E_\gamma = \bigcap_{n=1}^\infty \mathrm{cl}_S O_n$. Lemma 3.5 implies that for every $n \in \mathbb{N}$ there exists a countable set $N_n \subseteq I$ such that $\mathrm{cl}_S O_n = S \cap \pi^{-1}_{N_n}(\pi_{N_n}(\mathrm{cl}_S O_n))$. It is clear that the set $L(\gamma) = \bigcup_{n=1}^\infty N_n$ is countable and the following equality holds:

$$E_\gamma = S \cap \pi^{-1}_{L(\gamma)}(\pi_{L(\gamma)}(E_\gamma)).$$

Let $J_\gamma$ be a countable set in $I$ that contains $L_\gamma \cup L(\gamma)$ as a proper subset.

We verify that the families $\{J_\alpha : \alpha \leq \gamma\}$, $\{L_\alpha : \alpha \leq \gamma\}$, $\{F_\alpha : \alpha \leq \gamma\}$ and $\{E_\alpha : \alpha \leq \gamma\}$ satisfy conditions *i*)–*v*) for every $\alpha \leq \gamma$.

Conditions *i*), *ii*) and *iv*) follow immediately by the construction of $L_\gamma$, $J_\gamma$ and $F_\gamma$. Remember that $L(\gamma) \subseteq J_\gamma$, so condition *iii*) holds.

It only remains to verify condition $v$). Suppose there is a point $x \in E_\gamma \cap \overline{S}[J_\gamma \setminus L_\gamma]$. From the definition of the set $\overline{S}[J_\gamma \setminus L_\gamma]$ it follows that $\pi_i(x) = \pi_i(p)$ for every $i \in J_\gamma \setminus L_\gamma$. Lemma 3.8 implies that there exists $z \in S[L_\gamma]$ such that $\pi_{L_\gamma}(z) = \pi_{L_\gamma}(x)$. Then $\mathrm{supp}(z) \subseteq L_\gamma \subset J_\gamma$, hence $\pi_i(z) = \pi_i(p)$ for every $i \in J_\gamma \setminus L_\gamma$.

We have, thus, that $\pi_{J_\gamma}(z) = \pi_{J_\gamma}(x)$. Therefore

$$z \in S \cap \pi_{J_\gamma}^{-1}(\pi_{J_\gamma}(x)) \subseteq S \cap \pi_{J_\gamma}^{-1}(\pi_{J_\gamma}(E_\gamma)) = E_\gamma.$$

But the last fact implies that

$$f(z) \in f(E_\gamma \cap S[L_\gamma]) \subseteq F_\gamma \cap f(S[L_\gamma]) \neq \emptyset,$$

thus contradicting condition $ii$). This contradiction proves $v$).

Therefore, we can construct nested families $\mathcal{F} = \{F_\alpha : \alpha < \omega_1\}$, $\{E_\alpha : \alpha < \omega_1\}$, $\{J_\alpha : \alpha < \omega_1\}$ and $\{L_\alpha : \alpha < \omega_1\}$ that satisfy conditions $i$)–$v$) for every $\alpha < \omega_1$.

Since $K$ is compact, we have that $K \cap \bigcap \mathcal{F} \neq \emptyset$. Let $y^* \in K \cap \bigcap \mathcal{F}$ and $x^* \in f^{-1}(y^*)$. It is clear $x^* \in E_\alpha$ for every $\alpha < \omega_1$. Note that condition $iv$) implies that $\{J_\alpha \setminus L_\alpha : \alpha < \omega_1\}$ is a pairwise disjoint family of non-empty sets. Condition $v$) implies that $\mathrm{supp}(x^*) \cap (J_\alpha \setminus L_\alpha) \neq \emptyset$ for every $\alpha < \omega_1$, so the set $\mathrm{supp}(x^*)$ is uncountable. But this contradicts the fact that $x^* \in S \subseteq \Sigma(p)$. Hence there must exist a point $y \in K$ with $\chi(y, K) \leq \omega$. The theorem is proved. $\square$

**Corollary 4.2.** Let $X = \prod_{i \in I} X_i$ be a product of spaces such that $nw(X_i) \leq \omega$ for every $i \in I$, $p \in X$, $S \subseteq \Sigma(p)$ dense and almost invariant with respect to $p$. Let $f : S \to Y$ be a continuous map onto a regular space $Y$ of pointwise countable type. Then $nw(Y) \leq \omega$. In particular, if $Y$ is compact, then $Y$ is metrizable.

*Proof.* Clearly we can assume that $|I| > \omega$ and $|X_i| > 1$ for every $i \in I$. We claim that the set $Q = \{y \in Y : \chi(y, Y) \leq \omega\}$ is $G_\delta$-dense in $Y$. Indeed, let $P$ be a non-empty set of type $G_\delta$ in $Y$. We can assume that $P$ is closed in $Y$. Take a point $x \in P$. Since $Y$ is a pointwise countable type space, there exists a compact set $K \subseteq Y$ such that $x \in K$ and $\chi(K, Y) \leq \omega$. Then $C = P \cap K$ is a compact set such that $\chi(C, Y) \leq \omega$. By Theorem 4.1 there exists a point $y \in C$ such that $\chi(y, C) \leq \omega$. Therefore $\chi(y, Y) \leq \chi(y, C) \cdot \chi(C, Y) \leq \omega$. The claim is proved.

Theorem 3.6 implies that $nw(Q) \leq \omega$, in particular, $Q$ is a Lindelöf subspace of $Y$. Since $Q$ is $G_\delta$-dense in $Y$, Proposition 3.4 implies that $Y = Q$, hence $nw(Y) \leq \omega$. $\square$

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