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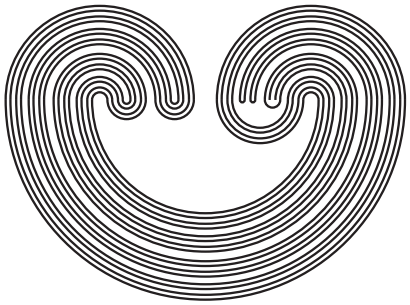

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ON GENERAL RESOLUTIONS OF GENERALIZED METRIC SPACES

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Abstract. We show that some kinds of generalized metric spaces are preserved by general resolutions due to Stephen Watson, under the criterion that a certain set Λ is F_σ-discrete in a space X.

1. Introduction

All spaces are assumed to be regular T_1 topological spaces. The letter $\mathbb{N}$ always denotes all positive integers and $\mathbb{R}$, $\mathbb{Q}$ all real, rational numbers, respectively. For a space X, let $\tau(X)$ denote the topology of X. For a set A, let $\mathcal{F}(A)$ be the totality of non-empty finite subsets of A. For families $\mathcal{U}_\alpha,\ \alpha \in A$, let $\bigwedge\{\mathcal{U}_\alpha \mid \alpha \in A\}$ be the family of all sets of the form $\bigcap\{U_\alpha \mid \alpha \in A\}$, where each $U_\alpha \in \mathcal{U}_\alpha,\ \alpha \in A$. For a family $\mathcal{U}$ of subsets of X, let $\Delta(\mathcal{U})$ be the totality of finite intersections of members of $\mathcal{U}$ and for $p \in X$ let $\mathcal{U}_p = \{U \in \mathcal{U} | p \in U\}$.

In this paper, we study the relation between classes of generalized metric spaces and their "general resolutions." The concept of general resolutions is due to S. Watson and it naturally generalizes special resolutions [6]. Especially, we show that some kinds of generalized metric spaces are preserved by general resolutions under some criterion. Here, we consider, as generalized metric spaces, the classes of spaces with a G_δ-diagonal, developable spaces, M_3-spaces

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and M_3-μ-spaces. As for the definitions and the fundamental properties of these classes, we refer the readers to [1]. As a study in the same direction, we note the paper of K. Richardson [4], where the preservation of metrizable spaces by general resolutions is discussed under the criterion that Λ, defined below, is F_σ-discrete in a space X. We show that this criterion also works well for the above generalized metric properties.

Definition 1.1 ([4, Definition 1])**.** Let X be a space and $\mathcal{A} = \{A_\alpha \mid \alpha \in I\}$ be a family of subsets of X. Let $\{f_\alpha \mid \alpha \in I\}$ be a collection of continuous mappings and $\{Y_\alpha \mid \alpha \in I\}$ a collection of spaces, where $f_\alpha : X \setminus A_\alpha \longrightarrow Y_\alpha$ for each $\alpha \in I$. For each $x \in X$, let $I(x) = \{\alpha \in I \mid x \in A_\alpha\}$ and let $Y_x = \prod\{Y_\alpha \mid \alpha \in I(x)\}$. If $I(x)$ is empty, then we take Y_x to be a singleton. We define a set Z as follows:

$$Z = \bigcup \big\{\{x\} \times Y_x \mid x \in X\big\}.$$

Let $\pi : Z \longrightarrow X$ be the projection onto X defined by $\pi(x, y) = x$ for each $(x, y) \in Z$. For each $\alpha \in I$, define the projection $\sigma_\alpha : Z \longrightarrow Y_\alpha$ by

$$\sigma_\alpha(x, y) = \begin{cases} y(\alpha) & \text{if } x \in A_\alpha, \\ f_\alpha(x) & \text{if } x \notin A_\alpha. \end{cases}$$

The general resolution Z has a topology the base of which consists of all sets of the form

$$\pi^{-1}(U) \cap \bigcap\{\sigma_\alpha^{-1}(V_\alpha) \mid \alpha \in J\},$$

where $U \in \tau(X)$, $V_\alpha \in \tau(Y_\alpha)$ for each $\alpha \in J$, and $J \subset I$ is finite. We represent Z constructed this way as

$$Z = R(X, \mathcal{A}, f_\alpha, Y_\alpha).$$

For latter use, we define the following operation $\otimes_\alpha$: Let $\alpha \in I$. For any subset A of X and B of Y_α, we define

$$A \otimes_\alpha B = \pi^{-1}(A) \cap \sigma_\alpha^{-1}(B).$$

Throughout this paper, we let

$$\Lambda = \bigcup\{A_\alpha \mid |Y_\alpha| > 1\}.$$

Then we call Λ *F_σ-discrete* in X if $\Lambda = \bigcup\{\Lambda(n) \mid n \in \mathbb{N}\}$, where each $\Lambda(n)$ is discrete and closed in X.

Finally, we note that any resolution $Z = R(X, \mathcal{A}, f_\alpha, Y_\alpha)$ of a regular space X in terms of regular spaces Y_α is regular, [5, Theorem 1]. We also note that in the sequel we use $I,\ I(x),\ \Lambda,\ \pi,\ \sigma_\alpha$ without any explanation.

2. Resolutions of generalized metric spaces.

Proposition 2.1. *Let $Z = R(X, \mathcal{A}, f_\alpha, Y_\alpha)$ have a G_δ-diagonal, where $\mathcal{A}$ is a family of closed subsets of a space X. Then $\Lambda = \bigcup\{\Lambda(n) \mid n \in \mathbb{N}\}$, where $\overline{\Lambda(n)} \setminus \Lambda = \emptyset$ for each $n \in \mathbb{N}$.*

Proof: Let $\{\mathcal{U}(n) \mid n \in \mathbb{N}\}$ be a G_δ-diagonal sequence for Z. For each $x \in \Lambda$, take $y_1,\ y_2 \in Y_x$ such that $(x, y_1) \neq (x, y_2)$. Then there exists $n(x) \in \mathbb{N}$ such that

$$(x, y_1) \notin S\left((x, y_2), \mathcal{U}(n(x))\right).$$

Let

$$\Lambda(n) = \{x \in \Lambda \mid n(x) = n\}.$$

Then $\Lambda = \bigcup_n \Lambda(n)$. Assume that for some n, $x \in \overline{\Lambda(n)} \setminus \Lambda$. Let $Y_x = \{a\}$. Since $\mathcal{U}(n)$ covers Z, there exists $U \in \mathcal{U}(n)$ such that $(x, a) \in U$. Then there exists $p \in \Lambda(n)$ such that $\{p\} \times Y_p \subset U$. This is a contradiction. □

The converse is not true as seen by Alexandroff double circle; that is, even if $X,\ Y_\alpha,\ \alpha \in I$, have a G_δ-diagonal and Λ satisfies the condition above, Z need not have a G_δ-diagonal.

Theorem 2.2. *Let $X,\ Y_\alpha,\ \alpha \in I$, have a G_δ-diagonal and let $\mathcal{A}$ be a point-countable closed cover of X. If Λ is F_σ-discrete in X, then $Z = R(X, \mathcal{A}, f_\alpha, Y_\alpha)$ has a G_δ-diagonal.*

Proof: Let $\{\mathcal{V}(n) \mid n \in \mathbb{N}\}$ be a G_δ-diagonal sequence for X, $\{\mathcal{W}(\alpha, m) \mid m \in \mathbb{N}\}$ a G_δ-diagonal sequence for Y_α for each $\alpha \in I$. Let $\Lambda = \bigcup\{\Lambda(n) \mid n \in \mathbb{N}\}$, where each $\Lambda(n)$ is discrete and closed in X. Let $n \in \mathbb{N}$ be fixed for a while. Take a family $\{U(p) \mid p \in \Lambda(n)\}$ of open subsets of X such that

$$U(p) \cap \Lambda(n) = \{p\},\ p \in \Lambda(n).$$

Since $\mathcal{A}$ is point-countable, for each $p \in \Lambda$ we can enumerate $I(p)$ as $I(p) = \{\alpha(p, i) \mid i \in \mathbb{N}\}$. For each $n,\ m,\ k \in \mathbb{N}$, define

$$\begin{aligned}\mathcal{U}(n,m,k) =&\{\pi^{-1}(X \setminus \Lambda(n))\} \\ &\cup \{U(p) \otimes_{\alpha(p,k)} W \mid W \in \mathcal{W}(\alpha(p,k),m), p \in \Lambda(n)\}.\end{aligned}$$

Enumerate

$$\{\mathcal{U}(n,m,k) \mid n,m,k \in \mathbb{N}\} \cup \{\pi^{-1}(\mathcal{V}(n)) \mid n \in \mathbb{N}\}$$

as $\{\mathcal{U}(n) \mid n \in \mathbb{N}\}$. We show that this forms a G_δ-diagonal sequence for Z. To see it, let $z_1 = (x_1, y_1)$, $z_2 = (x_2, y_2)$ be distinct points of Z. As the first case, let $x_1 = x_2 = x \in \Lambda(n)$ for some n. Then there exists $k \in \mathbb{N}$ such that $y_1(\alpha(x,k)) \neq y_2(\alpha(x,k)) \in Y_{\alpha(x,k)}$. Since $\{\mathcal{W}(\alpha(x,k),m) \mid m \in \mathbb{N}\}$ is a G_δ-diagonal sequence for $Y_{\alpha(x,k)}$, there exists m such that

$$y_1(\alpha(x,k)) \notin S(y_2(\alpha(x,k)), \mathcal{W}(\alpha(x,k),m)).$$

These imply

$$z_1 \notin S(z_2, \mathcal{U}(n,m,k)).$$

As the other case, let $x_1 \neq x_2$. Then there exists n such that $x_1 \notin S(x_2, \mathcal{V}(n))$, which implies

$$z_1 \notin S\left(z_2, \pi^{-1}(\mathcal{V}(n))\right). \qquad \square$$

The converse does not hold true, as seen by the following example:

Example 2.3. There exists a metric space X and a disjoint closed cover $\mathcal{A} = \{A_\alpha \mid \alpha \in I\}$ of X such that for each $\alpha \in I$ there exists a continuous mapping f_α of $X \setminus A_\alpha$ onto a compact metric space Y_α whose resolution $Z = R(X, \mathcal{A}, f_\alpha, Y_\alpha)$ is metrizable but Λ is not F_σ-discrete in X.

Proof: Let $X = \mathbb{R}$ with the usual topology. We take $\mathcal{A} = \{A_\alpha \mid \alpha \in I\}$ as follows:

$$I = \{0\} \cup (-\infty, -2) \cup (2, \infty)$$
$$A_0 = [-2,2], \quad A_\alpha = \{\alpha\} \ \ \mathrm{if}\ \alpha \in I \setminus \{0\}.$$

$\{f_\alpha \mid \alpha \in I\}$ is defined as follows: For each $\alpha \in I \setminus \{0\}$, f_α is a constant mapping of $X \setminus \{\alpha\}$ onto $Y_\alpha = \{0\}$ and $f_0 : X \setminus [-2,2] \longrightarrow Y_0 = [-2,2]$ is defined by

$$f_0(x) = \begin{cases} x-3 & \text{if } 2 < x \le 5 \\ 2 & \text{if } x > 5 \\ x+3 & \text{if } -5 \le x < -2 \\ -2 & \text{if } x < -5. \end{cases}$$

Then $\Lambda = A_0 = [-2, 2]$, and it is obviously not F_σ-discrete in X. It is also easy to see that the resolution $Z = R(X, \mathcal{A}, f_\alpha, Y_\alpha)$ has the base

$$\mathcal{B} = \big\{M\left((-2,1) : \varepsilon, \delta\right),\ M\left((2,-1) : \varepsilon, \delta\right) \Big| \varepsilon,\ \delta > 0\big\}$$
$$\cup\big\{N(z : \varepsilon, \delta)\Big| \varepsilon,\ \delta > 0,\ z \in A_0 \times Y_0 \setminus \{(2,-1),\ (-2,1)\}\big\}$$
$$\cup\big\{L\left((x,0) : \varepsilon\right) \Big| |x| > 2,\ \varepsilon > 0\big\},$$

where

$$M\left((-2,1) : \varepsilon, \delta\right) = \{(p,q) \in A_0 \times Y_0 \Big| -2 \le p < -2 + \varepsilon,$$
$$|q-1| < \delta\} \cup \{(p,0) \mid -2 - \varepsilon < x < -2\},$$

$$M\left((2,-1) : \varepsilon, \delta\right) = \{(p,q) \in A_0 \times Y_0 \Big| 2 - \varepsilon < p \le 2,\ |q+1| < \delta\}$$
$$\cup\{(p,0) \mid 2 < p < 2 + \varepsilon\},$$

$$L\left((x,0) : \varepsilon\right) = \{(p,0) \in Z \Big|\ |x-p| < \varepsilon,\ p \in X \setminus [-2,2]\}.$$

Then Z is metrizable because Z has a countable base

$$\{M\left((2,-1) : \frac{1}{n}, \frac{1}{m}\right),\ M\left((-2,1) : \frac{1}{n}, \frac{1}{m}\right) \mid n,\ m \in \mathbb{N}\}$$
$$\cup\big\{N(z : \frac{1}{n}, \frac{1}{m}) \Big|\ z \in \mathbb{Q} \times \mathbb{Q} \cap \left(A_0 \times Y_0 \setminus \{(2,-1),(-2,1)\}\right),$$
$$n,\ m \in \mathbb{N}\big\} \cup \{L\left((x,0) : \frac{1}{n}\right) \Big|\ |x| > 2,\ x \in \mathbb{Q},\ n \in \mathbb{N}\}. \quad \square$$

A space X is called a *developable* space if there exists a sequence $(\mathcal{U}(n))_n$ of open covers of X such that for each $x \in X$, $\{S(x,\mathcal{U}(n) \mid n \in \mathbb{N}\}$ is a local base at x in X. Such a sequence is called a development of X.

Theorem 2.4. *Let $X,\ Y_\alpha,\ \alpha \in I$, be developable spaces and let $\mathcal{A}$ be a point-countable closed cover of X. If Λ is F_σ-discrete in X, then $Z = R(X, \mathcal{A}, f_\alpha, Y_\alpha)$ is developable.*

Proof: Let $\Lambda = \bigcup\{\Lambda(n) \mid n \in \mathbb{N}\}$, where each $\Lambda(n)$ is discrete and closed in X. For each n, there exists a family $\{V(p)|p \in \Lambda(n)\}$ of open subsets of X such that

$$V(p) \cap \Lambda(n) = \{p\}, \quad p \in \Lambda(n).$$

Let $\{\mathcal{U}(m) \mid m \in \mathbb{N}\}$ be a development of X and for each $\alpha \in I$, let $\{\mathcal{V}(\alpha, k) \mid k \in \mathbb{N}\}$ be a development of Y_α. Since $\mathcal{A}$ is point-countable, for each $p \in \Lambda$, $I(p)$ is enumerated as follows:

$$I(p) = \{\alpha(p, i) \mid i \in \mathbb{N}\}.$$

For each $n,\ m \in \mathbb{N}$, let

$$\mathcal{U}(m,n) = \bigcup \{\mathcal{U}(m)_p \wedge \{V(p)\}|p \in \Lambda(n)\}.$$

For each $m,\ n,\ i,\ k \in \mathbb{N}$, define

$$\mathcal{W}(m,n,i,k) = \{\pi^{-1}(X \setminus \Lambda(n))\} \cup \{U \otimes_{\alpha(p,i)} V \mid U \in \mathcal{U}(m,n),$$
$$V \in \mathcal{V}(\alpha(p,i),k)),\ p \in \Lambda(n)\}.$$

Then it is an open cover of Z. Enumerate

$$\{\mathcal{W}(m,n,i,k) \mid m,n,i,k \in \mathbb{N}\} \cup \{\pi^{-1}(\mathcal{U}(n)) \mid n \in \mathbb{N}\}$$

as $\{\mathcal{W}(n) \mid n \in \mathbb{N}\}$. For each $\delta \in \mathcal{F}(\mathbb{N})$, let

$$\mathcal{W}(\delta) = \bigwedge \{\mathcal{W}(n) \mid n \in \delta\}.$$

We show that $\{\mathcal{W}(\delta) \mid \delta \in \mathcal{F}(\mathbb{N})\}$ is a development of Z.

Let $z = (x, y) \in G$, where G is open in Z. First, we assume the case $x \in \Lambda(n)$ for some n. There exist subsets $O \in \tau(X),\ W_\alpha \in \tau(Y_\alpha),\ \alpha \in J$ with $J \subset I$ finite such that

$$z \in \pi^{-1}(O) \cap \bigcap_{\alpha \in J} \sigma_\alpha^{-1}(W_\alpha) \subset G.$$

Since $\mathcal{A}$ is a family of closed subsets of X, without loss of generality we can assume $x \in A_\alpha$ for each $\alpha \in J$. Let $J = \{\alpha(x,i) \mid i \in N_0\}$,

where $N_0 \in \mathcal{F}(\mathbb{N})$. Since $(\mathcal{U}(m))_m$ is a development of X, there exists $m \in \mathbb{N}$ such that

$$S(x, \mathcal{U}(m)) \subset O \cap V(x).$$

For each $i \in N_0$, there exists $k(i) \in \mathbb{N}$ such that

$$S(y(\alpha(x,i)), \mathcal{V}(\alpha(x,i), k(i)) \subset W_{\alpha(x,i)}.$$

Let

$$\mathcal{W}(\delta) = \bigwedge\{\mathcal{W}(m,n,i,k(i)) \mid i \in N_o\},\ \delta \in \mathcal{F}(\mathbb{N}).$$

Then we have $S(z, \mathcal{W}(\delta)) \subset G$. Suppose the remaining case $x \notin \Lambda$. Then there exists $O \in \tau(X)$ such that $z \in \pi^{-1}(O) \subset G$. There exists $m \in \mathbb{N}$ such that $S(x, \mathcal{U}(m)) \subset O$. Then we have

$$S(z, \pi^{-1}(\mathcal{U}(m)) \subset G.$$

Hence, Z is developable. □

Theorem 2.5. *Let $X,\ Y_\alpha,\ \alpha \in I$, be M_3-spaces and let $\mathcal{A}$ be a point-countable closed cover of X. If Λ is F_σ-discrete in X, then $Z = R(X, \mathcal{A}, f_\alpha, Y_\alpha)$ is an M_3-space.*

Proof: Let $\Lambda = \bigcup\{\Lambda(n) \mid n \in \mathbb{N}\}$, where each $\Lambda(n)$ is discrete and closed in X. For each $n \in \mathbb{N}$, there exists a discrete open expansion $\{U(p) \mid p \in \Lambda(n)\}$ of $\big\{\{p\} \mid p \in \Lambda(n)\big\}$ in X. For each $p \in \Lambda(n)$, there exists a CP(=closure-preserving) closed neighborhood base $\mathcal{B}(p)$ of p in X such that $\bigcup \mathcal{B}(p) \subset U(p)$. Let $\bigcup\{\mathcal{B}(n) \mid n \in \mathbb{N}\}$ be a σ-CP closed quasi-base for X. For each $\alpha \in I$, let $\bigcup\{\mathcal{B}(\alpha, k) \mid k \in \mathbb{N}\}$ be a σ-CP closed quasi-base for Y_α. Since $\mathcal{A}$ is point-countable, for each $p \in \Lambda$, $I(p)$ is written as $I(p) = \{\alpha(p,m) \mid m \in \mathbb{N}\}$. For each $n,\ m,\ k \in \mathbb{N}$, define

$$\mathcal{B}(n,m,k) = \{B_1 \otimes_{\alpha(p,m)} B_2 \mid B_1 \in \mathcal{B}(p),\ B_2 \in \mathcal{B}(\alpha(x,m),k),\ p \in \Lambda(n)\}.$$

Let

$$\mathcal{G} = \bigcup\{\mathcal{B}(n,m,k) \mid n,m,k \in \mathbb{N}\} \cup \left(\bigcup\{\pi^{-1}(\mathcal{B}(n)) \mid n \in \mathbb{N}\}\right).$$

It is easily checked that $\mathcal{G}$ is a σ-CP in Z. To see that $\Delta\mathcal{G}$ is a quasi-base for Z, let $z = (x,y) \in O$, where O is open in Z. As the first case, suppose $p \in \Lambda(n),\ n \in \mathbb{N}$. There exist $U \in \tau(X),\ W_\alpha \in \tau(Y_\alpha),\ \alpha \in J \in \mathcal{F}(I)$, such that

$$z \in \pi^{-1}(U) \cap \left(\bigcap\{\sigma_\alpha^{-1}(W_\alpha) \mid \alpha \in J\}\right) \subset O.$$

Since $\mathcal{A}$ is a family of closed subsets of X, without loss of generality we can assume $p \in A_\alpha,\ \alpha \in J$. So, we can let $J = \{\alpha(p,m) \mid m \in N_0\}$, where $N_0 \in \mathcal{F}(\mathbb{N})$. Since $\bigcup_k \mathcal{B}(\alpha(p,m),k)$ is a quasi-base for $Y_{\alpha(p,m)}$, for each $m \in N_0$ there exists $B(\alpha(p,m)) \in \mathcal{B}(\alpha(p,m),k_m),\ k_m \in \mathbb{N}$, such that

$$y(\alpha(p,m)) \in \operatorname{Int} B(\alpha(p,m)) \subset B(\alpha(p,m)) \subset W_{\alpha(p,m)}.$$

Also, since $\mathcal{B}(p)$ is a neighborhood base of p in X, there exists $B_0 \in \mathcal{B}(p)$ such that

$$p \in \operatorname{Int} B_0 \subset B_0 \subset U \cap U(p).$$

Let

$$G = \pi^{-1}(B_0) \cap \bigcap \{\sigma^{-1}_{\alpha(p,m)}(B(\alpha(p,m))) \big| m \in N_0\}.$$

Then obviously we have $G \in \Delta(\mathcal{G})$ and $z \in \operatorname{Int} G \subset G \subset O$. As the remaining case, suppose $p \notin \Lambda$. Then obviously we can take $B_o \in \mathcal{B}(n),\ n \in \mathbb{N}$, such that

$$p \in \operatorname{Int} \pi^{-1}(B_0) \subset \pi^{-1}(B_0) \subset O.$$

Thus $\Delta(\mathcal{G})$ is a σ-CP quasi-base for Z, proving that Z is an M_3-space. □

A space X is called a *μ-space* if X is embedded into a countable product of paracompact F_σ-metrizable spaces. To use a characterization of M_3-μ-spaces, we repeat the definition of [3]: Let $\mathcal{U},\ \mathcal{F}$ be a family of subsets of a space X. $\mathcal{U}$ is said to be $\mathcal{F}$-preserving in both sides in X if for each $\mathcal{U}_0 \subset \mathcal{U}$ the following two conditions are satisfied:

(i) If $p \in X \setminus \bigcup \mathcal{U}_0$, then $p \in F \subset X \setminus \bigcup \mathcal{U}_0$ for some $F \in \mathcal{F}$;
(ii) if $\in \bigcap \mathcal{U}_0$, then $p \in F \subset \bigcap \mathcal{U}_0$ for some $F \in \mathcal{F}$.

It is known in [2] that an M_3-space X is a μ-space if and only if there exists a pair $\langle \bigcup_n \mathcal{U}(n), \bigcup_n \mathcal{F}(n) \rangle$ of families of subsets of X satisfying the following:

: (1) $\mathcal{U} = \bigcup \{\mathcal{U}(n) \mid n \in \mathbb{N}\}$ is a base for X such that each $\mathcal{U}(n)$ is $\mathcal{F}$-preserving in both sides in X;
: (2) $\mathcal{F} = \bigcup \{\mathcal{F}(n) \mid n \in \mathbb{N}\}$ is a network for X such that each $\mathcal{F}(n)$ is a discrete family of closed subsets of X.

In this case, we call the pair an *M-structure* for X.

Theorem 2.6. *Let $X,\ Y_\alpha,\ \alpha \in I$, be M_3-μ-spaces and let $\mathcal{A}$ be a point-countable closed cover of X. If Λ is F_σ-discrete in X, then $Z = R(X, \mathcal{A}, f_\alpha, Y_\alpha)$ is an M_3-μ-space.*

Proof: By the preceding theorem, Z is an M_3-space. Thus, it remains to show that Z has an M-structure. Let $\Lambda = \bigcup\{\Lambda(n) \mid n \in \mathbb{N}\}$, where each $\Lambda(n)$ is discrete and closed in X. For each n, there exists a discrete open expansion $\{U(p) \mid p \in \Lambda(n)\}$ of $\{\{p\} \mid p \in \Lambda(n)\}$ in X. Let $\langle \bigcup_n \mathcal{U}(n), \bigcup_n \mathcal{F}(n)\rangle$ be an M-structure for X and for each $\alpha \in I$ let $\langle \bigcup_i \mathcal{V}(\alpha, i), \bigcup_i \mathcal{F}(\alpha, i)\rangle$ be an M-structure for Y_α. For each $p \in \Lambda$, let $I(p) = \{\alpha(p, i) \mid i \in \mathbb{N}\}$. For each $n,\ m \in \mathbb{N},\ p \in \Lambda(n)$, let

$$\mathcal{U}(m, n : p) = \{U \in \mathcal{U}(m) \mid p \in U \subset U(p)\}.$$

Define $\mathcal{W}(m, n, i, k)$ and $\mathcal{H}(m, n, i, k)$ for each $m,\ n,\ i,\ k \in \mathbb{N}$ as follows:

$$\mathcal{W}(m, n, i, k : p) = \big\{U \otimes_{\alpha(p,i)} V \big| U \in \mathcal{U}(m, n : p),\ V \in \mathcal{V}(\alpha(p, i), k)\big\},\quad p \in \Lambda(n),$$

$$\mathcal{H}(m, n, i, k : p) = \big\{\{p\} \otimes_{\alpha(p,i)} F' \big|\ F' \in \mathcal{F}(\alpha(p, i), k)\big\},\ p \in \Lambda(n),$$

$$\mathcal{W}(m, n, i, k) = \bigcup\{\mathcal{W}(m, n, i, k : p) \mid p \in \Lambda(n)\},$$

$$\mathcal{H}(m, n, i, k) = \bigcup\{\mathcal{H}(m, n, i, k : p) | p \in \Lambda(n)\}.$$

Then it is easily checked that each $\mathcal{H}(m, n, i, k)$ is a discrete family of closed subsets of Z. Define

$$\mathcal{W}(m) = \pi^{-1}(\mathcal{U}(m)),\quad \mathcal{H}(m) = \pi^{-1}(\mathcal{F}(m)),\ m \in \mathbb{N}.$$

Enumerate

$$\{\mathcal{W}(m, n, i, k) \mid m,\ n,\ i,\ k \in \mathbb{N}\} \cup \{\mathcal{W}(m) \mid m \in \mathbb{N}\}$$

as $\{\mathcal{V}(n) \mid n \in \mathbb{N}\}$, and enumerate

$$\{\mathcal{H}(m, n, i, k) \mid m,\ n,\ i,\ k \in \mathbb{N}\} \cup \{\mathcal{H}(m) \mid m \in \mathbb{N}\}$$

as $\{\mathcal{L}(n) \mid n \in \mathbb{N}\}$. For each $\delta \in \mathcal{F}(\mathbb{N})$, define

$$\mathcal{V}(\delta) = \bigwedge\{\mathcal{V}(n) \mid n \in \delta\},\ \mathcal{L}(\delta) = \bigwedge\{\mathcal{L}(n) \mid n \in \delta\}.$$

Then it is easy to see that

$$\big\langle \bigcup\{\mathcal{V}(\delta) \mid \delta \in \mathcal{F}(\mathbb{N})\}, \bigcup\{\mathcal{L}(\delta) \mid \delta \in \mathcal{F}(\mathbb{N})\}\big\rangle$$

is an M-structure for Z. □

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