Topology Proceedings

Web: http://topology.auburn.edu/tp/
Mail: Topology Proceedings
Department of Mathematics & Statistics
Auburn University, Alabama 36849, USA
E-mail: topolog@auburn.edu
ISSN: 0146-4124

COPYRIGHT © by Topology Proceedings. All rights reserved.

TOPOLOGY
PROCEEDINGS
Volume 28, No.2, 2004
Pages 467-478

TOTALLY BOUNDED GROUP TOPOLOGIES THAT ARE BOHR TOPOLOGIES OF LCA GROUPS

JORGE GALINDO*

Dedicated to Wis Comfort

Abstract. Some conditions that are obviously necessary for a totally bounded group to be the Bohr group of a locally compact Abelian group are shown to be sufficient.

1. Introduction

The importance of compactifications in the analysis and applications of topological spaces can hardly be overemphasized. It suffices to think of one-point or Stone-Čech compactifications. For topological spaces carrying some additional structure (algebraic or geometric, for instance) compactifications are usually sought to somehow reflect that structure. In the case of a topological group G this goal may be achieved by considering its *Bohr compactification* bG. This compactification appears as one of the best behaved and most useful compactifications for topological groups. Even if its significance varies depending on the class of topological groups in question (it may even be totally useless), there is a big range of groups for which it is a really useful tool.

2000 *Mathematics Subject Classification.* 54H11, 22A05.

Key words and phrases. Bohr topology, locally compact Abelian group, k-extension, totally bounded.

*Research supported by European Union (FEDER) and the Spanish Ministry of Science and Education, grant number MTM2004-07665-C02-01 and by Generalitat Valenciana, grant number 04I189.01/1.

The Bohr compactification reaches its full richness in the case of locally compact Abelian groups (LCA groups for short). An LCA group $(G,\mathcal{T})$ may always be identified with a dense subgroup of its Bohr compactification by means of a continuous group isomorphism. The topology that G inherits from $b(G,\mathcal{T})$ is however different from $\mathcal{T}$ unless G is compact. This topology is the so-called *Bohr topology* of $(G,\mathcal{T})$ and we shall denote it by $\mathcal{T}^+$.

The Bohr topology is always totally bounded because $(G,\mathcal{T}^+)$ is a subgroup of the compact group $b(G,\mathcal{T})$. After the essential observation of Comfort and Ross [1] we know that every totally bounded group topology on an Abelian group is the weak topology $\mathcal{T}_H$ induced by some group of characters H of G. It is quite easy to see that for an Abelian topological group G we have the equality $G^+ = (G,\mathcal{T}_{\widehat{G}})$ where $\widehat{G}$ refers to the group of *continuous* characters of G.

Many papers in the literature try to relate properties of a topological group G with properties of G^+ as a way to test if the *simpler* object G^+ can be faithfully used to analyze the more complicated object G. In this paper we take a different, in some sense opposite, point of view: departing from a totally bounded group topology $\mathcal{T}_H$ on an Abelian group G (so that H is a subgroup of $\operatorname{Hom}(G,\mathbb{T}))$ we try to discern when there is a locally compact group topology $\mathcal{T}$ on G such that $(G,\mathcal{T}^+) = (G,\mathcal{T}_H)$. Let us put this question formally:

Question 1. If G is an Abelian group and $\mathcal{T}_b$ is a totally bounded group topology on G, give a criterion to determine if there is some locally compact topology $\mathcal{T}$ on G such that $\mathcal{T}^+ = \mathcal{T}_b$.

The first difficulty is to identify the right locally compact topology candidate to solve our problem. To that end we may use a theorem of Trigos-Arrieta (that depends strongly on Glicksberg's theorem [7]), Theorem 1.1 below, that identifies the k-extension of G^+ with G, for locally compact G. Recall that the k-extension of a topology $\mathcal{T}$ is the strongest topology $k\mathcal{T}$ with the property that $\mathcal{T}$ and $k\mathcal{T}$ produce the same compact subsets.

To avoid some cumbersome notation we shall write kG instead of $(G,k\mathcal{T})$ when no confusion seems possible and, in the same spirit, G^+ instead of $(G,\mathcal{T}^+)$, so that, for instance, the expression $k(G^+)$ should be understood as $(G,k(\mathcal{T}^+))$.

Theorem 1.1 ([13]). *If G is an LCA group then $k(G^+) = G$.*

Our strategy to determine when a totally bounded group topology $\mathcal{T}_H$ (H being a subgroup of $Hom(G, \mathbb{T})$) on an Abelian group G is the Bohr topology of a locally compact topology $\mathcal{T}$ will consist in studying first the k-extension $k\mathcal{T}_H$. This latter topology must be locally compact and when this is the case we shall try to discriminate whether $\mathcal{T}_H$ is the Bohr topology of $(G, k\mathcal{T}_H)$ i.e. whether $(G, k\mathcal{T}_H)\widehat{\ } = H$.

This way of facing Question 1 leads us to the paper [3] where Comfort, Trigos and Wu posed the following question

Question 2. [3] Characterize those totally bounded groups G such that kG is locally compact and $G = (kG)^+$.

The remarks above based on Theorem 1.1 show that questions 1 and 2 are in fact equivalent. Question 2 has indeed been the main inspiration of this paper.

2. When is kG locally compact?

It is not easy to find conditions that ensure that the k-extension of a totally bounded group is locally compact. We find here that some of the obviously necessary conditions are also sufficient.

It should be remarked that kG might not be the right object to deal with questions 1 and 2 for the simple reason that kG need not be a topological group for some topological groups G (see [9] as cited in [13, Example 1.14]). The next observation is a reformulation of Ellis theorem on joint continuity of multiplication [6] and indicates that this does not happen when kG is locally compact.

Lemma 2.1. *Let $(G, \mathcal{T})$ be a topological group. If $kG = (G, k\mathcal{T})$ has a compact subset K with nonvoid interior, then kG is also a topological group.*

Proof. For every topological group $(G, \mathcal{T})$ inversion is $k\mathcal{T}$-continuous and addition is separately $k\mathcal{T}$-continuous, see [11, Theorem 1.7].

Let x_0 belong to the interior of K. If $g \in G$, then $g - x_0 + K$ (translations are continuous) will be a compact $k\mathcal{T}$-neighbourhood of g and kG turns to be locally compact. We can now apply Ellis theorem ([6, Theorem 2]) asserting that a locally compact topological space endowed with a separately continuous addition must be a topological group. □

2.1. **Connected groups.** A connected LCA group without compact subgroups is always topologically isomorphic to a vector group $\mathbb{R}^n$ of dimension n. Its family of compact subsets has therefore an increasing countable cofinal (with respect to inclusion) subfamily whose elements can be chosen to be homeomorphic to the cube $\mathbb{I}^n$. We shall refer to this property by saying that G is hemicompact with decomposition $G = \cup_m K_m$, K_m homeomorphic to the cube $\mathbb{I}^n$. This turns to be a sufficient condition for the k-extension of a totally bounded group to be locally compact.

Lemma 2.2. *The k-extension kG of a totally bounded group G is a connected locally compact group without compact subgroups (i.e. topologically isomorphic to $\mathbb{R}^n$) if and only if G is hemicompact with decomposition $G = \cup_m K_m$ and K_m homeomorphic to some cube $\mathbb{I}^n$.*

Proof. Only one direction needs some proof. Suppose that G admits the above decomposition $G = \cup_m K_m$. Let V_1 denote the interior of K_1 regarded as a subset of $\mathbb{R}^n$. We will show that V_1 is open in kG. Observe that in virtue of Lemma 2.1 kG will automatically be a locally compact topological group.

Since kG is a k-space, and because of the hemicompactness property, it will suffice to prove that V_1 is open in K_m for every m (note that $V_1 \subseteq K_1 \subseteq K_m$). But V_1 is an open subset of $\mathbb{R}^n$ and K_m is also homeomorphic to a cube contained in $\mathbb{R}^n$. By Brouwer's invariance of domain theorem [5, Theorem 3.1] we deduce that V_1 must also be open in the copy of $\mathbb{R}^n$ that contains K_m, and hence V_1 must be open as a subset of K_m. We thus have that K_1 has nonvoid interior V_1 and by Lemma 2.1, kG is a connected locally Euclidean topological group. As such kG will be topologically isomorphic to $\mathbb{R}^m \times \mathbb{T}^k$, for some integers m and k [8, 9.8]. Being the increasing union of cubes kG must be simply connected and we deduce that $k = 0$. Since V_1 is open in $\mathbb{R}^n$ we conclude that G must in fact be topologically isomorphic to $\mathbb{R}^n$. □

2.2. **Totally disconnected groups.** A locally compact totally disconnected group G always contains some compact open subgroup K (many, indeed). K will also be a compact subgroup of G^+ and G^+/K will carry the Bohr topology of the discrete group G/K. This is therefore a necessary condition for a totally bounded group to be the Bohr group of a totally disconnected locally compact group. The next result is the first step in proving that it is also sufficient.

Lemma 2.3. *Let G denote a totally bounded group. If K is a compact subgroup of G such that every character of G/K is continuous, then K is open in kG. kG is therefore locally compact when G contains such a subgroup K.*

Proof. Let K be a subgroup of G such that every character of G/K is continuous. The quotient G/K is a totally bounded group and hence its topology is the topology of pointwise convergence on its group of characters, that in this case is the group of *all* characters of G/K. As is well known such a group has no infinite compact subsets (this is a particular case of Glicksberg's theorem, see [10] for an older proof of the discrete case). From this fact we may easily conclude that kG/K is discrete (and hence that K is open in kG): simply consider an arbitrary function f defined on kG/K, since kG/K has no infinite compact subsets, the composition of f with the quotient mapping $p\colon G \to G/K$, $f \circ p$, is continuous on compact subsets and is therefore kG-continuous. This shows that kG/K is discrete. □

3. When does $\widehat{G} = \widehat{kG}$ holds?

The path we have chosen to answer Question 1 comprises finding conditions for the k-extension kG of a given totally bounded group G to be locally compact, to subsequently require the Bohr topology of kG to be exactly the topology of G. This happens exactly when the k-extension of $\mathcal{T}$ adds no new continuous character, that is, when $\widehat{G} = \widehat{kG}$.

3.1. Totally bounded topologies on vector spaces.

We begin with a straightforward Lemma that we will use a couple of times.

Lemma 3.1. *Let $\mathcal{T}_{H_1} \subset \mathcal{T}_{H_2}$ be two totally bounded group topologies on a group G (H_1 and H_2 are two groups of characters of G). If there is a $\mathcal{T}_{H_1}$-closed subgroup K of G such that the restriction of both topologies to K coincide and such that every $\mathcal{T}_{H_2}$-continuous character that vanishes on K is necessarily $\mathcal{T}_{H_1}$-continuous (i.e. both topologies induce the same topology on G/K), then $\mathcal{T}_{H_1} = \mathcal{T}_{H_2}$.*

Proof. It is enough to observe that every $\mathcal{T}_{H_2}$-continuous character $\chi \in H_2$ can be written as the sum of a $\mathcal{T}_{H_1}$-continuous character (the $\mathcal{T}_{H_1}$-continuous extension to G of $\chi_{|_K}$) and a character vanishing on K, the latter factorizes trough G/K and is therefore also $\mathcal{T}_{H_1}$-continuous. □

Lemma 3.2. *Let $\mathcal{T}_H$ be a totally bounded group topology on $\mathbb{R}$ weaker than the usual Euclidean topology $\mathcal{T}_u$. If for some nontrivial cyclic subgroup $\langle x\rangle$ of $\mathbb{R}$, all characters are $\mathcal{T}_H$-continuous, then*

$$(\mathbb{R}, \mathcal{T}_u)\widehat{\ } = H = (\mathbb{R}, \mathcal{T}_H)\widehat{\ } .$$

Proof. Let us recall first that continuous characters of $(\mathbb{R}, \mathcal{T}_u)\widehat{\ }$ are in bijective correspondence with elements of $\mathbb{R}$. To every $r \in \mathbb{R}$ there corresponds the character χ_r defined by

$$\chi_r(t) = \exp(2\pi \mathrm{i}\, rt) \text{ for all } t \in \mathbb{R}.$$

By hypothesis all characters of $\langle\, x \,\rangle$ are $\mathcal{T}_H$-continuous and thus the restriction of the character $\chi_{\frac{1}{2x}}$ to $\langle\, x \,\rangle$ will coincide with the restriction of some character $\chi_v \in H$. Since χ_v and $\chi_{1/(2x)}$ agree on $\langle\, x \,\rangle$ we have that $vx - 1/2 \in \mathbb{Z}$ which means that $0 \neq 2v = z_0/x$ for some integer $z_0 \in \mathbb{Z}$.

Let now χ_s be any continuous character of $(\mathbb{R}, \mathcal{T}_u)$. Choose $s_1 \in \mathbb{R}$ with $z_0 s_1 = s$. Since the restriction of χ_{s_1} to $\langle\, x \,\rangle$ is continuous, there is some real number w with $\chi_w \in H$ and such that $s_1 x - wx = n$ for some $n \in \mathbb{Z}$. It is now readily verified that [1]

$$\chi_s = \chi_{z_0\, s_1} = \chi_{z_0\left(\frac{n}{x}+w\right)} = \chi_{\frac{z_0\, n}{x}} + \chi_{z_0\, w} = 2n\,(\chi_v) + z_0\chi_w$$

hence $\chi_s \in H$ for every $s \in \mathbb{R}$. □

The following theorem shows that the question in the title of this Section may be answered by examining one small finite subset of $\mathbb{R}^n$. As in the case of Lemma 3.2 we can associate to every element $\mathbf{t} = (t_1, \ldots, t_n) \in \mathbb{R}^n$ a continuous character of $(\mathbb{R}^n, \mathcal{T}_u)$

$$\chi_{\mathbf{t}}(s_1, \ldots, s_n) = \exp\left[2\pi \mathrm{i}\left(\sum_{j=1}^{m} t_j \cdot s_j\right)\right]$$

and this correspondence establishes an isomorphism between elements of $\mathbb{R}^n$ and the character group of $(\mathbb{R}^n, \mathcal{T}_u)$.

[1] We use additive notation for characters so that $(\chi_1 + \chi_2)(x) = \chi_1(x) \cdot \chi_2(x)$.

Let us also agree to denote by $\mathbf{e}_i$ the i-th vector of the canonical basis of $\mathbb{R}^n$, that is $\mathbf{e}_i = (\overbrace{0, \ldots, 0, 1}^{(i)}, 0, \ldots, 0)$

Theorem 3.3. *Let $\mathcal{T}_H$ be a totally bounded group topology on $\mathbb{R}^n$ weaker than the usual Euclidean topology $\mathcal{T}_u$. If the subgroup generated by some $\mathbb{R}$-basis of the form $\{s_1\mathbf{e}_1, \ldots, s_n\mathbf{e}_n\}$ has no $\mathcal{T}_H$-discontinuous characters, then*

$$(\mathbb{R}^n, \mathcal{T}_u)\widehat{\ } = H = (\mathbb{R}^n, \mathcal{T}_H)\widehat{\ } .$$

Proof. We want to proceed by induction on n, the following claim will pave our way to that end.

Claim. *Let V_1 denote the closed one dimensional vector subspace of $(\mathbb{R}^n, \mathcal{T}_u)$ generated by $\mathbf{e}_1$. For any given $t_2, \ldots, t_n \in \mathbb{R}$ there is a $\mathcal{T}_H$-continuous character χ such that*

$$\chi(V_1) = \{1\} \qquad \textit{and} \qquad \chi(s_i\mathbf{e}_i) = \exp(2\pi\mathrm{i}\, t_i), \quad i = 2, \ldots, n.$$

Arguing as in the first part of Lemma 3.2 but starting with a character that takes $s_1\mathbf{e}_1$ to -1 and vanishes on $s_i\mathbf{e}_i$ we find $\mathbf{r} = (\frac{m_1}{s_1}, r_2, \ldots, r_n) \in \mathbb{R}^n$ with

$$0 \neq m_1 \in \mathbb{Z}, \qquad r_i s_i \in \mathbb{Z} \qquad \text{and} \qquad \chi_{\mathbf{r}} \in H.$$

Considering next a character of $\langle s_1\mathbf{e}_1, \ldots, s_n\mathbf{e}_n\rangle$ sending $s_1\mathbf{e}_1$ to 1 and $s_i\mathbf{e}_i$ to $\exp(2\pi\mathrm{i}t_i/m_1)$ we find a $\mathcal{T}_H$-continuous character $\chi_{\mathbf{t}}$ with

$$\mathbf{t} = \left(\frac{z_1}{s_1}, \frac{t_2 + z_2 m_1}{s_2\, m_1}, \ldots, \frac{t_n + z_n m_1}{s_n\, m_1}\right)$$

where $z_i \in \mathbb{Z}$. The character $\chi_0 = m_1\chi_{\mathbf{t}} - z_1\chi_{\mathbf{r}}$ is therefore also $\mathcal{T}_H$-continuous and has the form

$$\chi_0 = \left(0, \frac{t_2 + z_2 m_1}{s_2} - z_1 r_2, \ldots, \frac{t_n + z_n m_1}{s_n} - z_1 r_n.\right).$$

Clearly, χ_0 is the claimed character.

Lemma 3.2 proves that the present Theorem holds when $n = 1$. We now proceed by induction on n and assume as inductive hypothesis that it also holds on $\mathbb{R}^{n-1}$.

Consider the quotient $\mathbb{R}^n/V_1 \cong \mathbb{R}^{n-1}$. The quotient topology that $\mathcal{T}_H$ induces on $\mathbb{R}^n/V_1$ is obviously weaker than the Euclidean topology of $\mathbb{R}^{n-1}$. It will suffice to check that every character ψ of the lattice $\langle s_2\mathbf{e}_2, \ldots, s_n\mathbf{e}_n\rangle + V_1$ extends to a continuous character of $(\mathbb{R}^n, \mathcal{T}_H)/V_1$. Applying the Claim with t_i such that $\psi(s_i\mathbf{e}_i + V_1) = \exp(2\pi \mathrm{i} t_i)$ we find $\chi \in H$ vanishing on V_1 with $\chi(s_i\mathbf{e}_i) = \psi(s_i\mathbf{e}_i + V_1)$. This character χ clearly induces a continuous character of $(\mathbb{R}^n, \mathcal{T}_H)/V_1$ that coincides with ψ on $\langle s_2\mathbf{e}_2, \ldots, s_n\mathbf{e}_n\rangle + V_1$. Since ψ was arbitrary, we may apply our induction hypothesis to conclude that all characters of $(\mathbb{R}^{n-1}, \mathcal{T}_u) = (\mathbb{R}^n, \mathcal{T}_u)/V_1$ are continuous as characters of $(\mathbb{R}^n, \mathcal{T}_H)/V_1$ or, what is the same, that all characters of $(\mathbb{R}^n, \mathcal{T}_u)$ that are trivial on V_1 are $\mathcal{T}_H$-continuous. Since by Lemma 3.2 V_1 has the Bohr topology of $(\mathbb{R}, \mathcal{T}_u)$ we deduce from Lemma 3.1 that all characters of $(\mathbb{R}^n, \mathcal{T}_u)$ are $\mathcal{T}_H$-continuous. □

The following observation will help to put together a characterization for connected groups.

Lemma 3.4. *If K is a compact subgroup of a topological group G, then $k(G/K) = kG/K$.*

Proof. We shall first prove that, regardless of the properties of K, it is always true that the identity mapping $i\colon kG/K \to k(G/K)$ is continuous. Denote by $p\colon G \to G/K$ the quotient mapping and let L be any compact subset of G. The subset $p(L)$ is compact in G/K and all three topologies, G/K, kG/K and $k(G/K)$ agree on $p(L)$. Thus $i \circ p$ has a continuous restriction to the arbitrarily chosen compact subset L and is a continuous function on the k-space kG. Since kG/K has the final topology induced by $p\colon kG \to kG/K$, the continuity of i follows.

The inverse $i^{-1}\colon k(G/K) \to kG/K$ is continuous as well. This follows directly from [8, 5.24(a)] where it is proven that a compact subset of G/K is always the image under $p\colon G \to G/K$ of some compact subset of G, provided K is compact. □

Corollary 3.5. *Let G be a connected totally bounded group. The following are equivalent:*

(1) *kG is a LCA group with $(kG)^+ = G$.*
(2) *The group G has a maximal compact subgroup K such that: (a) G/K is hemicompact with decomposition $G/K = \cup_m K_m$, K_m homeomorphic to some cube $\mathbb{I}^n$ and (b) G/K contains a lattice (subgroup generated by n linearly independent vectors) L all whose characters are G/K-continuous.*

Proof. It is proved in [13, Corolary 2.6] that kG must be connected if it is locally compact and $(kG)^+$ is connected. Therefore 1 implies that kG contains a maximal compact subgroup K with kG/K topologically isomorphic to $\mathbb{R}^n$ for some n. Since lattices are discrete subgroups it is then clear that (1) implies (2).

If (2) holds we know by Lemma 2.2 that $k(G/K)$ is locally compact. Since K is compact we may apply Lemma 3.4 to see that kG is locally compact as well. Now $k(G/K)$ is connected and has no compact subgroups, it will therefore be topologically isomorphic to some vector group $\mathbb{R}^n$. Theorem 3.3 finally shows that $(kG)^+/K = G^+/K$ and (1) follows via Lemma 3.1. □

3.2. **The totally disconnected case.** We see here that the condition described in Lemma 2.3 is strong enough to completely answer our question.

Theorem 3.6. *Let $G = (G, \mathcal{T}_H)$ be a totally disconnected, totally bounded group. The following assertions are equivalent:*

(1) *kG is a locally compact group with $(kG)^+ = G$.*
(2) *G contains a compact subgroup K such that every character of G/K is continuous.*
(3) *G contains a compact subgroup K such that every character that vanishes on K is automatically continuous, in symbols*

$$\chi \in Hom(G, \mathbb{T}) \text{ and } \chi(K) = \{1\} \Longrightarrow \chi \in H\,.$$

Proof. If kG is a locally compact totally disconnected group with $(kG)^+ = G$, kG will contain a compact open subgroup K. Since kG/K is then discrete (2) follows.

Conditions (2) and (3) are obviously equivalent and we now prove that (3) implies (1).

If we assume (3), Lemma 2.3 proves that K is open in kG (here we have actually used (2)). To see that $G = kG^+$ we simply apply Lemma 3.1 to G and K. Since K is kG-compact, it will inherit the same topology from kG, G and $(kG)^+$. This together with Condition (3) make again the hypothesis of Lemma 3.1. □

4. Concluding remarks

This note in a whole is admittedly only a first approximation to questions 1 and 2. What we have pursued here is to find some conditions that may help to identify when a totally bounded group topology is the Bohr topology of an LCA group. It is certainly apparent that the conditions appearing in Corollary 3.5 and Theorem 3.6 do not contain the simplest (nor the deepest) imaginable reasons for a totally bounded group to be the Bohr topology of an LCA group. In that direction we leave a first step untouched, to characterize when (or find significant sufficient conditions for) a totally bounded group has the Bohr topology of a discrete group. Some properties that such groups must possess are already known: they cannot have infinite compact subsets [7] and must be zero-dimensional [2] or [4] (even strongly zero-dimensional by the results of [12]). It plainly suffices to know that *all* characters are continuous to realize that a totally bounded group has the Bohr topology of a discrete group, but this is just another way of posing the same question, even if this seems easier than identifying discontinuous characters in the non-discrete case.

Let us try to summarize here the sufficient conditions noticed in this paper.

Theorem 4.1. *Let G be a totally bounded group.*

(1) *Assume that G is totally disconnected. kG is locally compact and $(kG)^+ = G$ if and only if G has a compact subgroup such that every character of G vanishing on K is continuous.*
(2) *Assume G is connected. kG is locally compact and $(kG)^+ = G$ if and only G has a maximal compact subgroup K such that: (a) G/K is hemicompact with decomposition $G = \cup_m K_m$, K_m homeomorphic to some cube $\mathbb{I}^n$ and (b) G/K contains a lattice L (finitely generated subgroup with $(G/K)/L$ compact) with the property that all characters of G/K have a continuous restriction to L.*
(3) *kG is locally compact and $(kG)^+ = G$ if and only if the connected component G_0 of G satisfies condition* (2) *and G contains a compact subgroup K whose projection $(K + G_0)/G_0 \subseteq G/G_0$ satisfies condition* (1) *(every character of G/G_0 vanishing on $(K + G_0)/G_0$ is continuous).*

Proof. Statements (1) and (2) have been proven in Theorem 3.6 and Corollary 3.5, respectively. It is easy to deduce from Statement (3) that G_0 is locally compact as a subgroup of kG and $(G_0)^+ = (kG_0)^+$. Arguing as in Lemma 2.3 one finds that kG/G_0 is a locally compact group and, since G_0 also is, that kG is locally compact. Lemma 3.1 applied to the quotient $kG/(K+G_0)$ finally shows that $(kG)^+ = G$. □

The author would like to thank the referee for a good number of suggestions and recommendations that doubtless have made the paper more transparent (and sometimes coherent).

References

[1] W. W. Comfort and K. A. Ross, *Topologies induced by groups of characters*, Fund. Math. **55**:283–291, 1964.

[2] W. W. Comfort and F. Javier Trigos-Arrieta, *Remarks on a theorem of Glicksberg*, In: General topology and applications (Staten Island, NY, 1989), pages 25–33. Dekker, New York, 1991.

[3] W. W. Comfort, F. Javier Trigos-Arrieta, and Ta Sun Wu, *The Bohr compactification, modulo a metrizable subgroup*, Fund. Math., **143**(2):119–136, 1993.

[4] Eric K. van Douwen, *The maximal totally bounded group topology on G and the biggest minimal G-space, for abelian groups G*, Topology Appl., **34**(1):69–91, 1990.

[5] James Dugundji. *Topology.* Allyn and Bacon Inc., Boston, Mass., 1966.

[6] Robert Ellis, *Locally compact transformation groups*, Duke Math. J., **24**:119–125, 1957.

[7] Irving Glicksberg, *Uniform boundedness for groups*, Canad. J. Math., **14**:269–276, 1962.

[8] Edwin Hewitt and Kenneth A. Ross, *Abstract harmonic analysis. Vol. I: Structure of topological groups. Integration theory, group representations.* Academic Press Inc., Publishers, New York, 1963.

[9] W. F. Lamartin, *On the foundations of k-group theory*, Dissertationes Math. (Rozprawy Mat.), **146**:32, 1977.

[10] Horst Leptin, *Zur Dualitätstheorie projektiver Limites abelscher Gruppen*, Abh. Math. Sem. Univ. Hamburg, **19**:264–268, 1955.

[11] N. Noble, *k-groups and duality*, Trans. Amer. Math. Soc., **151**:551–561, 1970.

[12] Dmitrii B. Shakhmatov, *Imbeddings into topological groups preserving dimensions*, Topology Appl., **36**(2):181–204, 1990. Seminar on General Topology and Topological Algebra (Moscow, 1988/1989).

[13] F. Javier Trigos-Arrieta, *Continuity, boundedness, connectedness and the Lindelöf property for topological groups*, J. Pure Appl. Algebra, **70**:199–210, 1991.

Departamento de Matemáticas, Universitat Jaume I, 8029-AP Castellón, Spain
E-mail address: jgalindo@mat.uji.es
URL: http://www3.uji.es/~jgalindo