Topology Proceedings

Web:	http://topology.auburn.edu/tp/
Mail:	Topology Proceedings Department of Mathematics & Statistics Auburn University, Alabama 36849, USA
E-mail:	topolog@auburn.edu
ISSN:	0146-4124

TOPOLOGY PROCEEDINGS
Volume 29, No. 1, 2005
Pages 415-420

DILEMMA IN TOPOLOGY (AND IN SCIENCE): BIZARRE VS. COMMON

A. LELEK

This is the third time in my life that I am giving a talk[1] about the history of mathematics [5]. The first time was when I was a student. The second time it happened, I was invited, in 1968, to participate in the ceremony bestowing an honorary membership of the Polish Mathematical Society on Bronisław Knaster. For the ceremony, a large crowd of students and faculty of the University of Wrocław had gathered in the beautiful original baroque Leopoldine Auditorium, in the main university building, on the left bank of the Oder River, in Wrocław, Poland. The contents of my talk were published afterwards in *Wiadomosci Matematyczne* **11** (1969), 81–86.

Knaster is famous for his discovery, in 1922, of the so-called hereditarily indecomposable continuum, later named the *pseudo-arc* [3]. Indecomposable continua had been discovered as early as 1910 by L. E. J. Brouwer. For years, they have been considered bizarre counter-examples. An indecomposable continuum is a continuum which cannot be represented as the union of two proper subcontinua. You cannot break it: it is like an atom, or, better yet, a nucleus, or an elementary particle. The points are indecomposable. Still, there exist such indecomposable objects which are bigger than points. Today, we know that they are not so bizarre or uncommon. They come to existence, in a natural way, as attractors in dynamical systems, closely related to the theory of partial differential equations. They are closer to the real physical world

[1]This paper is based on a talk delivered at the annual meeting of the American Mathematical Society on January 15, 1993 [5].

than they seemed in the years 1910, 1920, etc. The continuum constructed and studied by Knaster is even more paradoxical: it is bigger than a point, it is indecomposable, and each continuum contained in it is also indecomposable. That was very hard to imagine. But that was not the end of the story. Some new paradoxical properties of Knaster's continuum were scheduled to follow. It turns out that this object is, in a sense, cyclic and acyclic at the same time. Cyclic objects are those that have holes in them, like the circle or the sphere, i.e., the surface of the Euclidean ball. The holes make it possible to rotate the objects in a certain way so that each given point can be moved, by the rotation of the whole thing, to the location of any other given point. We call such objects *homogeneous*. More precisely, there should exist, for each pair of points, a homeomorphism transforming the object onto itself and sending the first point into the second one. In 1948, R. H. Bing proved that Knaster's continuum is homogeneous [1]; so it is cyclic, or behaves like a circle, in some way. Yet, also in 1948, Edwin E. Moise showed that Knaster's continuum is homeomorphic to each of its non-degenerate subcontinua [6]. That is not at all the property of a circle; it is the property of an arc. So Knaster's continuum behaves like an arc, or it is acyclic. For this reason, it has been called the pseudo-arc.

Knaster himself did not mind being associated with bizarre and exotic things. He shared with us that paradoxical, strange, and unusual phenomena had always interested him more than regular, common, and usual ones. He went further to say that the distinction of being bizarre is often a result of illusion, and the things which look strange on the first sight turn out to be more important, more natural, and even, at the end, after a closer look, more common than those which look "normal." Maybe it is a joke nature plays on us. Maybe it is a by-product of our specific relationship to nature as human beings with our idiosyncrasies and flaws of perception.

A totally different point of view was represented by Karol Borsuk. To him, paradoxical phenomena existed only to be recognized, then classified, and ultimately, eliminated. Borsuk was the author of two major theories in general topology: the theory of shapes, originated in the 1960s, and the theory of retracts, which he began developing in the 1930s. The retracts and, particularly, the absolute retracts, or the ARs, were meant to be the most regular, nice, and good objects of research in topology. Borsuk's book *Theory of Retracts* [2] reflects this philosophy.

Borsuk saw the universe also as composed of atoms, or elementary particles. The objects there were like polyhedra composed of simplexes. So the ARs had been created to imitate the behavior of simplexes and Euclidean cells. The cells themselves were too restrictive and too special. Even among the ARs, however, some paradoxes had been found, and Borsuk devoted quite a lot of effort to study, classify, and eliminate them finally. I, myself, contributed to this study [2, p. 148].

Despite the difference of opinion, there was no animosity between Borsuk and Knaster. The two men did not see one another very often. Knaster preferred to stay in Wrocław, and Borsuk was busy in Warsaw. But there was a constant exchange of ideas in letters and through students who traveled back and forth. The first time I heard Borsuk speak was when he came to Wrocław and gave a talk at Knaster's seminar. His talk was meticulously organized, well prepared, and polished. Borsuk's desire for smoothness and lack of singularities was reflected in his style of lecturing. Knaster's style was more personal. He included jokes and stories, frequently repeating himself. People knew about it and smiled, but nevertheless, had been visibly influenced by them.

The most influential mathematician in Wrocław, however, was not Knaster. It was Hugo Steinhaus. As it happened, mathematics in Wrocław inherited traditions and people from two pre-World War II mathematical centers in Poland, namely, Warsaw and Lwów. Knaster was from Warsaw; Steinhaus was from Lwów. Steinhaus used to say that his greatest discovery in mathematics had been Stefan Banach (1892-1945). These and other pieces of history are described in Stan Ulam's book, *Adventures of a Mathematician* [7].

In the early spring of 1952, when I was almost at the end of my first year at the University, I received a message that Professor Steinhaus wanted to see me. I went to his house which was located in a suburban area of the city. It was a two-story house comprised of two apartments, and the Steinhauses lived in the apartment on the second floor. I learned later that the apartment on the first floor was occupied by Knaster and his family. I rang the bell and Steinhaus opened the door. He led me to his room, where he was typing. I looked at the typewriter and noticed that it was mathematics. There were also scissors and glue. Steinhaus told me that when you write mathematics, you have to know how to cut and splice. He proceeded to inform me that I had been chosen to be the recipient of that year's Stefan Banach prize for the best freshman in mathematics. He told me that these prizes had been given every year by Mrs. Banach in memory of her husband who died several years earlier. Steinhaus gave me the money. It was 100 złotys. He then asked if I would like him to type a thank-you note to Mrs. Banach. He took his paper he was typing out of the typewriter and typed the note which I signed. I was, of course, elated to get the distinction of being recognized as the best student in a body of approximately 70 students. Steinhaus, however, diminished the impact. "You know," he said, "that in algebra each group has to have a distinguished element, called the unity or the neutral element. Well, we had to find one in this group of students, and you have been selected to be a neutral element." I felt like I was beginning to be bizarre.

Before I left Steinhaus's apartment, I looked outside the window and there I saw a tall, bald man tending a garden. I did not know who it was at the time, but a couple of years after that, I realized it was Knaster. (Gardening was one of his most favorite pastimes.)

I was destined to have a lot of interaction in mathematics with Knaster in the years which followed, and although I had little interaction with Steinhaus during the same period, it did produce some interesting results. (His suggestion that the 2-cell admits uncountably many disjoint Peano parametrizations was important to my conclusions presented in [4].)

All this took place in Wrocław, just north of the Sudeten Mountains in southwestern Poland. But for more than two centuries, the University of Wrocław had been a part of the German city of Breslau, the main and old metropolitan spot in Lower Silesia, a city that was devastated in 1945 as World War II came to a close.

When we were there, in the 1950s, most of the classes and meetings were held in the dark and heavy buildings of the Polytechnic School which survived the assault. On Fridays, at 5:00, the meetings of the Wrocław division of the Polish Mathematical Society were held in Room 105. I remember one day, at the start of the meeting, Steinhaus said, "Gentlemen, Albert Einstein died last week. I ask you to stand and honor him with one minute of silence."

On another occasion, he made a comment which impressed me deeply. It was a presentation in applied mathematics, an area sought after and supported. The talk was on aerodynamics, and Steinhaus gave a short speech about complex numbers. He explained that for a long time after their first appearance, complex numbers had been considered an oddity, a bizarre product of the curious minds of mathematicians. Only through the advancement of science in engineering was an application found in a very practical and concrete realm which everybody was able to appreciate. There exist problems in aerodynamics, we were told, related to aviation that cannot be solved at all without using complex numbers: they turn out to be completely indispensable. "Remember, gentlemen," Steinhaus said, "without complex numbers, man would not fly."

In 1970, I decided to leave Poland and move to America. It was a long and complicated journey, with extended stops in Turkey and Sweden. I guess I continued to do things in a bizarre way. When I finally arrived in the United States and someone would ask me why I settled in Texas, I would recall my time in Wrocław (and southwestern Poland) and answer that I was addicted to the Southwest

Sometimes, listening to lectures on complex analysis, I think of the pseudo-arc and other strange objects and topics in the theory of continua. Are they really that strange and bizarre, or do they only seem to be that way due to circumstances? A connection with the philosophy of science arises, as there likely is a degree of universality in this question.

References

[1] R. H. Bing, "A homogeneous indecomposable plane continuum," *Duke Math. J.* **15** (1948), no. 3, 729–742.

[2] Karol Borsuk, *Theory of Retracts.* Monografie Matematyczne, Tom 44. Warsaw: PWN, 1967.

[3] Bronisław Knaster, "Un continu dont tout sous-continu est indécomposable," *Fund. Math.* **3** (1922), 247–286.

[4] A. Lelek, "Disjoint mappings and the span of spaces," *Fund. Math.* **55** (1964), 199–214.

[5] ———, "Dilemma in topology (and in science): Bizarre vs. common." American Mathematical Society National Meeting, Special Session on History of General Topology, I. San Antonio Convention Center, San Antonio, Texas. 15 Jan. 1993.

[6] Edwin E. Moise, "An indecomposable plane continuum which is homeomorphic to each of its nondegenerate subcontinua," *Trans. Amer. Math. Soc.* **63** (1948), no. 3, 581–594.

[7] S. M. Ulam, *Adventures of a Mathematician.* New York: Charles Scribner's Sons, 1976.

Department of Mathematics and Statistics; Auburn University; Auburn University, AL 36849

E-mail address: `leleka@dms.auburn.edu`