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ON CLOSED IMAGES OF SEQUENTIALLY MESOCOMPACT SPACES

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ABSTRACT. In this paper, we give some characterizations of sequentially mesocompact spaces. By these characterizations, we prove that closed sequentially quotient mappings preserve sequentially mesocompact spaces. As some applications of this result, we show that sequential mesocompactness is preserved under both closed countable-to-one mappings (with regular domain) and closed finite-to-one mappings, which answers a question posed by S.Lin.

1. INTRODUCTION

In his paper[13], V.J.Mancuso claimed that perfect mappings preserve sequential mesocompactness, but his claim was not true. J.R.Boone[1] noticed the error, and gave a example to show that the perfect image of a Tychonoff sequentially mesocompact space need not be sequentially mesocompact. Then, which closed mappings preserve sequential mesocompactness? J.R.Boone[2] proved that closed presequential mappings preserve normal sequential mesocompactness. It is a natural question whether the condition "normal" in the foregoing result can be omitted? That is, we have the following question.

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Question 1.1. Is sequential mesocompactness preserved under closed presequential mappings?

In addition, note that perfect mappings do not preserve sequential mesocompactness. S.Lin [10] raised following question in a survey on "spaces and mappings".

Question 1.2. [10]. Is sequential mesocompactness preserved under closed finite-to-one mappings?

In this paper, we give some characterizations of sequentially mesocompact spaces. By these characterizations, we prove that closed sequentially quotient mappings preserve sequential mesocompactness, which answers the above Question 1.1. As some applications of this result, we show that sequential mesocompactness is preserved under both closed countable-to-one mappings (with regular domain) and closed finite-to-one mappings, which answers the above Question 1.2.

Throughout this paper, all spaces are assumed to be Hausdorff and all mappings are assumed to be continuous and onto. N denotes the set of all natural numbers. For a set A, $|A|$ denotes the cardinality of A. Let $\mathcal{U}$ be a family of subsets of a space X. $\mathcal{U}^F$ denotes the family consisting of all finite unions of sets from $\mathcal{U}$. $\bigcup\mathcal{U}$ and $\bigcap\mathcal{U}$ denote the union $\bigcup\{U : U \in \mathcal{U}\}$ and the intersection $\bigcap\{U : U \in \mathcal{U}\}$ respectively. For $A \subset X$ and $x \in X$, $(\mathcal{U})_A$ denotes the family $\{U \in \mathcal{U} : U \bigcap A \neq \emptyset\}$ and $(\mathcal{U})_{\{x\}}$ is replaced by $(\mathcal{U})_x$, $st(A,\mathcal{U})$ denotes the union $\bigcup(\mathcal{U})_A = \bigcup\{U \in \mathcal{U} : U \bigcap A \neq \emptyset\}$ and $st(\{x\},\mathcal{U})$ is replaced by $st(x,\mathcal{U})$. Let $f : X \longrightarrow Y$ be a mapping, and let $\mathcal{U}$ and $\mathcal{V}$ are two families of subsets of X and Y respectively, then $f(\mathcal{U}) = \{f(U) : U \in \mathcal{U}\}$ and $f^{-1}(\mathcal{V}) = \{f^{-1}(V) : V \in \mathcal{V}\}$. For definitions of concepts and terms used without definition below, see [5].

Definition 1.3. A family $\mathcal{U}$ of subsets of a space X is called *cs*-finite[13] (compact-finite[13], point-finite[8]), if for every convergent sequence $L \subset X$ (every compact subset $K \subset X$, every point $x \in X$), $(\mathcal{U})_L$ ($(\mathcal{U})_K$, $(\mathcal{U})_x$) is finite; is called cs^*-finite, if for every convergent sequence $L \subset X$, there exists a subsequence S of L such that $(\mathcal{U})_S$ is finite.

Remark 1.4. It is obvious that compact-finite $\Longrightarrow$ *cs*-finite $\Longrightarrow$ cs^*-finite $\Longrightarrow$ point-finite.

Definition 1.5. A space X is called sequentially mesocompact[13] (mesocompact[13], metacompact[8]), if every open cover of X has a cs-finite (compact-finite, point-finite) open refinement.

Remark 1.6. It is clear that mesocompact $\Longrightarrow$ sequential mesocompact $\Longrightarrow$ metacompact from Definition 1.5. Note that perfect mappings preserve mesocompactness[9] and metacompactness[15], and perfect mappings do not preserve sequentially mesocompactness[1]. So none of the implications can be reversed.

Definition 1.7. Let $\mathcal{U}$ and $\mathcal{V}$ be two families of subsets of a space X.

(1) We say that $\mathcal{V}$ is a partial refinement of $\mathcal{U}$, if for every $V \in \mathcal{V}$ there is $U \in \mathcal{U}$ such that $V \subset U$; moreover, we say that $\mathcal{V}$ is a refinement of $\mathcal{U}$, if in addition $\bigcup \mathcal{V} = \bigcup \mathcal{U}$ is also satisfied.

(2) We say that $\mathcal{V}$ is a cs^*-wise W-refinement of $\mathcal{U}$, if $\bigcup \mathcal{V}=\bigcup \mathcal{U}$ and for every convergent sequence $L \subset X$, there exists a subsequence S of L such that $(\mathcal{V})_S$ is a partial refinement of some finite subfamily $\mathcal{U}'$ of $\mathcal{U}$.

(3) We say that $\mathcal{V}$ is a pointwise W-refinement of $\mathcal{U}$[8], if $\bigcup \mathcal{V}=\bigcup \mathcal{U}$ and for every point $x \in X$, $(\mathcal{V})_x$ is a partial refinement of some finite subfamily $\mathcal{U}'$ of $\mathcal{U}$.

(4) We say that $\mathcal{U}$ is subsequence-refined in X, if for every convergent sequence L in X, there exists a subsequence S of L such that $S \subset U$ for some $U \in \mathcal{U}$.

(5) We say that $\mathcal{U}$ is directed[8] if for all $U \in \mathcal{U}$ and $V \in \mathcal{U}$, there exists $W \in \mathcal{U}$ such that $U \bigcup V \subset W$.

Remark 1.8. It is obvious that cs^*-wise W-refinement $\Longrightarrow$ pointwise W-refinement.

Definition 1.9. Let H be a subset of a space X. H is called sequentially closed[6], if no sequence in H converges to a point not in H. A space X is called sequential, if every sequentially closed subset of X is closed.

Definition 1.10. Let $f : X \longrightarrow Y$ be a mapping.

(1) f is called closed Lindelof[11], if f is closed and $f^{-1}(y)$ is a Lindelof subset of X for every $y \in Y$.

(2) f is called presequential[2], if for every convergent sequence $\{y_n : n \in N\}$ in Y, $y_n \longrightarrow y$, which is not eventually equal to y, $\bigcup\{f^{-1}(y_n) : n \in N, y_n \neq y\}$ is not sequentially closed.

(3) f is called sequentially quotient[4], if for every convergent sequence L in Y, there exists a convergent sequence S in X such that $f(S)$ is a subsequence of L.

Remark 1.11. It is known that a mapping $f : X \longrightarrow Y$ is sequentially quotient provided: H is sequentially closed in Y if and only if $f^{-1}(H)$ is sequential closed in X[3] (also see [12]).

2. Main Results

We start by giving some lemmas.

Lemma 2.1. *Let $\{\mathcal{U}_n\}$ be a sequence of open covers of X such that for every $n \in N$, $\mathcal{U}_{n+1}$ is a cs^*-wise W-refinement of $\mathcal{U}_n$. Then $\mathcal{U}_1$ has an open refinement $\mathcal{V} = \bigcup_{n=2}^{\infty} \mathcal{V}_n$ such that $\mathcal{V}_{n+1}$ is cs^*-finite for every $n \in N$. In particular, if every open cover of X has an open cs^*-wise W-refinement, then every open cover of X has an open refinement $\mathcal{V} = \bigcup_{n=1}^{\infty} \mathcal{V}_n$ such that $\mathcal{V}_n$ is cs^*-finite for every $n \in N$.*

Proof. Note that $\mathcal{U}_{n+1}$ is a pointwise W-refinement of $\mathcal{U}_n$ for every $n \in N$. By [8, Proposition 2.2], $\mathcal{U}_1$ has an open refinement $\mathcal{V} = \bigcup_{n=2}^{\infty} \mathcal{V}_n$ such that for every $n \in N$ and $A \subset X$, if $(\mathcal{U}_{n+1})_A$ is a partial refinement of a subfamily $\mathcal{U}'$ of $\mathcal{U}_n$, then $|(\mathcal{V}_{n+1})_A| \leq |\mathcal{U}'|$. We claim that $\mathcal{V}_{n+1}$ is cs^*-finite for every $n \in N$. In fact, let L be a convergent sequence. Since $\mathcal{U}_{n+1}$ is a cs^*-wise W-refinement of $\mathcal{U}_n$, there exists a subsequence S of L and a finite subfamily $\mathcal{U}'$ of $\mathcal{U}_n$ such that $(\mathcal{U}_{n+1})_S$ is a partial refinement of $\mathcal{U}'$. So $|(\mathcal{V}_{n+1})_S| \leq |\mathcal{U}'| < \infty$, that is, $\mathcal{V}_{n+1}$ is cs^*-finite. □

Lemma 2.2. *Let $\{\mathcal{U}_n\}$ be a sequence of cs^*-finite families of subsets of a space X, and let L be a convergent sequence. Then there exists a subsequence S of L such that $(\mathcal{U}_n)_S$ is finite for every $n \in N$.*

Proof. We construct S by induction as follows.

Since $\mathcal{U}_1$ is cs^*-finite, there exists a subsequence L_1 of L such that $(\mathcal{U}_1)_{L_1}$ is finite. Let x_1 be the first term of L_1. Similarly, since $\mathcal{U}_2$ is cs^*-finite, there exists a subsequence L_2 of L_1 such that $(\mathcal{U}_2)_{L_2}$ is finite. Let x_2 be the second term of L_2. Suppose $x_1, x_2, \cdots, x_k$ and $L_1, L_2, \cdots, L_k$ have been obtained in such a way. Since $\mathcal{U}_{k+1}$ is cs^*-finite, there exists a subsequence L_{k+1} of L_k such that $(\mathcal{U}_{k+1})_{L_{k+1}}$ is finite. Let x_{k+1} be the $(k+1)$-*th* term of L_{k+1}. Thus we construct a subsequence $S = \{x_n : n \in N\}$ of L by induction. Note that $S \subset L_n \bigcup \{x_1, x_2, \cdots, x_{n-1}\}$ for every $n \in N$. Since $\mathcal{U}_n$ is point-finite and $(\mathcal{U}_n)_{L_n}$ is finite for every $n \in N$, $(\mathcal{U}_n)_S$ is finite for every $n \in N$. □

Lemma 2.3. *Let $\mathcal{U}$ be a family of subsets of a space X. Then $\mathcal{U}$ is cs-finite if and only if it is cs*-finite.*

Proof. The necessity is clear. We will prove the sufficiency. Let $\mathcal{U}$ be cs^*-finite. If $\mathcal{U}$ is not cs-finite, then there exists a convergent sequence $L = \{x_n : n \in N\}$ in X such that $(\mathcal{U})_L$ is infinite. We construct y_k for every $k \in N$ by induction as follows.

Write $n_1 = 1$, put $y_1 = x_{n_1}$. Since $(\mathcal{U})_{x_{n_1}}$ is finite, there exists $n \in N$ such that $(\mathcal{U})_{x_n} - (\mathcal{U})_{x_{n_1}} \neq \emptyset$. Let $n_2 = \min\{n \in N : (\mathcal{U})_{x_n} - (\mathcal{U})_{x_{n_1}} \neq \emptyset\}$. Then $n_2 > n_1$. Put $y_2 = x_{n_2}$. Suppose $y_1, y_2, \cdots, y_k$ have been constructed in such a way. Since $(\mathcal{U})_{\{y_1, y_2, \cdots, y_k\}}$ is finite, there exists $n \in N$ such that $(\mathcal{U})_{x_n} - (\mathcal{U})_{\{y_1, y_2, \cdots, y_k\}} \neq \emptyset$. Let $n_{k+1} = \min\{n \in N : (\mathcal{U})_{x_n} - (\mathcal{U})_{\{y_1, y_2, \cdots, y_k\}} \neq \emptyset\}$. It is easy to see that $n_1 < n_2 < \cdots < n_k < n_{k+1}$ from the selection of $n_1, n_2, \cdots, n_k, n_{k+1}$. Put $y_{k+1} = x_{n_{k+1}}$. Thus we construct a subsequence $S = \{y_k : k \in N\}$ of L by induction. S is a convergent sequence. It is not difficult to prove that $(\mathcal{U})_{S'}$ is infinite for every subsequence S' of S. This is contradictory of the fact that $\mathcal{U}$ is cs^*-finite. □

In the proof of the following theorem, we use the technique invented by H.J.K. Junnila[8].

Theorem 2.4. *For any space X, the following are equivalent.*

(1) X is sequentially mesocompact.

(2) Every open cover of X has a cs-finite open refinement.*

(3) Every directed open cover of X has a closure-preserving closed refinement which is subsequence-refined in X.

(4) Every open cover of X has an open cs-wise W-refinement.*

Proof. (1)$\Longleftrightarrow$(2) from Lemma 2.3.

(2)$\Longrightarrow$(3): Let $\mathcal{U}$ be a directed open cover of X. Then $\mathcal{U}$ has a cs^*-finite open refinement $\mathcal{V}$. Let $F(U) = \{x \in X : st(x, \mathcal{V}) \subset U\}$. Put $\mathcal{F} = \{F(U) : U \in \mathcal{U}\}$. By the proof of (ii)$\Longrightarrow$(i) in [8, Lemma 2.3], $\mathcal{F}$ is a closure-preserving closed refinement of $\mathcal{U}$.

Let L be a convergent sequence. Since $\mathcal{V}$ is cs^*-finite, there exists a subsequence S of L such that $(\mathcal{V})_S$ is finite. Note that $\mathcal{U}$

is directed. Thus $st(S,\mathcal{V}) \subset U$ for some $U \in \mathcal{U}$, so $S \subset F(U)$. This proves that $\mathcal{F}$ is subsequence-refined in X.

(3)$\Longrightarrow$(4): Let $\mathcal{U}$ be an open cover of X. Since X is metacompact from [8, Theorem 3.1], $\mathcal{U}$ has a point-finite open refinement $\mathcal{V}$. As a directed open cover of X, $\mathcal{V}^F$ has a closure-preserving closed refinement $\mathcal{F}$ which is subsequence-refined in X. Let $W_x = \bigcap(\mathcal{V})_x - \bigcup(\mathcal{F} - (\mathcal{F})_x)$ for every $x \in X$ and put $\mathcal{W} = \{W_x : x \in X\}$. That $\mathcal{W}$ is an open cover of X is obvious. For every $F \in \mathcal{F}$, we pick a finite subfamily $\mathcal{V}_F$ of $\mathcal{V}$ such that $F \subset \bigcup \mathcal{V}_F$. Then $(\mathcal{W})_F$ is a partial refinement of $\mathcal{V}_F$. In fact, if $W_x \in (\mathcal{W})_F$, that is $W_x \bigcap F \neq \emptyset$, then $x \in F$ from the definition of W_x. So there exists $V \in \mathcal{V}_F \subset \mathcal{V}$ such that $x \in V$, thus $W_x \subset \bigcap(\mathcal{V})_x \subset V$.

Let L be a convergent sequence. Then there exists a subsequence S of L and $F \in \mathcal{F}$ such that $S \subset F$. Note that $(\mathcal{W})_S \subset (\mathcal{W})_F$. $(\mathcal{W})_s$ is a partial refinement of a finite subfamily $\mathcal{V}_F$ of $\mathcal{V}$. Thus $\mathcal{W}$ is an open cs^*-wise W-refinement of $\mathcal{V}$ as well as of $\mathcal{U}$.

(4)$\Longrightarrow$(2): Let $\mathcal{U}$ be an open cover. By Lemma 2.1 and Lemma 2.2, $\mathcal{U}$ has an open refinement $\mathcal{W} = \bigcup_{n=1}^{\infty} \mathcal{W}_n$ such that for every convergent sequence L, there exists a subsequence S of L such that $(\mathcal{W}_n)_S$ is finite for every $n \in N$. Put $G_n = \bigcup_{k=1}^{n}(\bigcup \mathcal{W}_k)$ for every $n \in N$. As an open cover of X, $\mathcal{G} = \{G_n : n \in N\}$ has an open cs^*-wise W-refinement $\mathcal{A}$. Put $F_0 = \emptyset$, and for every $n \in N$, put $F_n = \{x \in X : st(x, \mathcal{A}) \subset G_n\}$ and $\mathcal{V}_n = \{W - F_{n-1} : W \in \mathcal{W}_n\}$. Similar to the proof of (ii)$\Longrightarrow$(i) in [8, Lemma 2.3], it is easily seen that $\mathcal{V} = \bigcup_{n=1}^{\infty} \mathcal{V}_n$ is an open refinement of $\mathcal{U}$. So it suffices to show that $\mathcal{V}$ is cs^*-finite.

Let L be a convergent sequence. Then there exists a subsequence L' of L such that $(\mathcal{W}_n)_{L'}$ is finite for every $n \in N$. Since $\mathcal{A}$ is a cs^*-wise W-refinement of $\mathcal{G}$, there exists a subsequence S of L' such that $(\mathcal{A})_S$ is a partial refinement of some finite subfamily of $\mathcal{G}$. Note that $\mathcal{G}$ is directed. There exists $n_0 \in N$ such that $st(S, \mathcal{A}) \subset G_{n_0}$, hence $S \subset F_{n_0}$. Thus S is a subsequence of L, and $|(\mathcal{V})_S| \leq \Sigma_{n=1}^{\infty} |(\mathcal{V}_n)_S| = \Sigma_{n=1}^{n_0} |(\mathcal{V}_n)_S| \leq \Sigma_{n=1}^{n_0} |(\mathcal{W}_n)_S| \leq \Sigma_{n=1}^{n_0} |(\mathcal{W}_n)_{L'}| < \infty$. So $\mathcal{V}$ is cs^*-finite. □

Now we give the main theorem of this paper.

Theorem 2.5. *Let $f : X \longrightarrow Y$ be a closed sequentially quotient mapping. If X is sequentially mesocompact, then Y is sequentially mesocompact.*

Proof. Let $\mathcal{V}$ be a directed open cover of Y. Then $\mathcal{U} = f^{-1}(\mathcal{V})$ is a directed open cover of X. By Theorem 2.4(3), $\mathcal{U}$ has a closure-preserving closed refinement $\mathcal{F}$ which is subsequence-refined in X. Since f is closed, $f(\mathcal{F})$ is a closure-preserving closed refinement $\mathcal{V}$. Let L be a convergent sequence in Y. Since f is sequentially quotient, there exists a convergent sequence S in X such that $f(S)$ is a subsequence of L. Since $\mathcal{F}$ is subsequence-refined in X, there exists a subsequence S' of S such that $S' \subset F$ for some $F \in \mathcal{F}$. Thus $f(S')$ is a subsequence of L, and $f(S') \in f(F) \in f(\mathcal{F})$. This proves that $f(\mathcal{F})$ is subsequence-refined in Y. By Theorem 2.4(3), Y is sequentially mesocompact. □

Recall that a mapping $f : X \longrightarrow Y$ is subsequence-covering [7] if for every convergent sequence S in Y, there is a compact subset K in X such that $f(K)$ is an infinite subsequence (containing its limit point) of S. By [7, Proposition 2.1], we have the following corollary.

Corollary 2.6. *Let $f : X \longrightarrow Y$ be a closed subsequence-covering mapping, where points in X are G_δ. If X is sequentially mesocompact, then Y is sequentially mesocompact.*

In [2], J.R.Boone proved that presequential mappings are sequentially quotient[2, Theorem 6]. In fact, the implication is reversible.

Proposition 2.7. *Let $f : X \longrightarrow Y$ be a mapping. Then f is presequential if and only if it is sequentially quotient.*

Proof. We only need to prove the sufficiency.

Let $\{y_n : n \in N\}$ be a convergent sequence in Y, $y_n \longrightarrow y$, which is not eventually equal to y. Since f is a sequentially quotient mapping, there exists a convergent sequence $\{x_k\}$ in X, $x_k \longrightarrow x$, such that $\{f(x_k)\} = \{y_{n_k}\}$ is a subsequence of $\{y_n\}$. Let $\{x_{k_i}\}$ be a subsequence of $\{x_k\}$ such that $f(x_{k_i}) = y_{n_{k_i}} \neq y$. Write $x_{k_i} = a_i$ and $y_{n_{k_i}} = z_i$ for every $i \in N$. Then $\{z_i\}$ is a convergent sequence in Y, $z_i \longrightarrow y$, and a_i is a sequence in $\bigcup\{f^{-1}(z_i) : i \in N, z_i \neq y\}$, $a_i \longrightarrow x$. Note that $f(x) = y$. $x \notin \bigcup\{f^{-1}(z_i) : i \in N, z_i \neq y\}$, so $\bigcup\{f^{-1}(z_i) : i \in N, z_i \neq y\}$ is not sequentially closed. Thus f is presequential. □

The following corollary is immediate from Proposition 2.7 and Theorem 2.5. It improves [2, Theorem 8] by omitting "normal" in [2, Theorem 8].

Corollary 2.8. *The closed presequential image of a sequentially mesocompact space is a sequentially mesocompact space.*

3. Some applications

Now we give some applications of Theorem 2.5. At first, we have the following two lemmas. Lemma 3.1(1) is well known, and it is implicit already in the old result of Lubben that, for a perfect mapping $f : X \to Y$, $f^{-1}(K)$ is compact in X whenever K is compact in Y.

Lemma 3.1. *(1) Every perfect mapping is compact-covering (also see* [9]*, for example).*

(2) Every closed Lindelof mapping with regular domain is compact-covering[11, Lemma 2.1]*.*

(3) Every countable-to-one compact-covering mapping is sequentially quotient[3, Proposition 1.1 and Proposition 1.3].

Recall a space X is said to have point-G_δ property, if every point in X is a G_δ-subset of X.

Lemma 3.2. [12, Lemma 2.2.3]. *Let $f : X \longrightarrow Y$ be a closed mapping. If one of the following holds, then f is sequentially quotient.*

(1) X is a sequential space.

(2) X is a regular space with point-G_δ property.

The following theorem is immediate from Lemma 3.1, Lemma 3.2 and Theorem 2.5.

Theorem 3.3. *Let $f : X \longrightarrow Y$ be a closed mapping, X be sequentially mesocompact. If one of the following holds, then Y is sequentially mesocompact.*

(1) f is a finite-to-one mapping.

(2) f is a countable-to-one mapping with regular domain.

(3) X is a sequential space.

(4) X is a regular space with point-G_δ property.

Remark 3.4. Theorem 3.3(1) answers a question posed by S.Lin in [10], but we do not know if the condition "regular" in Theorem 3.3(2) can be omitted.

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