Topology Proceedings

Web: http://topology.auburn.edu/tp/
Mail: Topology Proceedings
Department of Mathematics & Statistics
Auburn University, Alabama 36849, USA
E-mail: topolog@auburn.edu
ISSN: 0146-4124

TOPOLOGY PROCEEDINGS
Volume 31, No. 1, 2007
Pages 77-87
http://topology.auburn.edu/tp/

METRIZABLE FIBERINGS OF CONTINUOUS HAUSDORFF IMAGES OF COMPACT ORDERED SPACES

D. DANIEL

Abstract. A Tychonoff space A is metrizably fibered if and only if there exists a continuous map $F : A \to B$ onto a metrizable space B such that for each $b \in B$, $F^{-1}(b)$ is metrizable. It has been shown that every first countable Hausdorff continuous image of the lexicographic square is metrizably fibered. We show here that the techniques used in that result may be modified to show that every first countable Hausdorff space X that is the continuous image of a metrizably fibered compact ordered space is metrizably fibered.

1. Introduction

In [14], Vladimir V. Tkachuk studies metrizably fibered compact spaces, those compact spaces that admit a continuous mapping onto a metrizable space with metrizable point inverses. Among the questions raised by Tkachuk is whether every first countable continuous Hausdorff image of the lexicographic square is metrizably fibered. This was answered affirmatively in [3]. A related question is whether each first countable continuous image of a metrizably fibered compactum is metrizably fibered. This latter question remains open but here we demonstrate that the techniques utilized

2000 *Mathematics Subject Classification.* Primary: 54F15; Secondary: 54C05, 54F05, 54F50.

Key words and phrases. lexicographic square, metrizably fibered, ordered continuum.

in [3] may be modified to show that the result holds in the class of first countable Hausdorff continuous images of metrizably fibered compact ordered spaces. In particular, every first countable Hausdorff space X that is the continuous image of a metrizably fibered compact ordered space is metrizably fibered.

Other interesting papers in the area of metrizably fibered spaces and related spaces include [1], [5], and [13].

2. Preliminaries

A Tychonoff space A is *metrizably fibered* if and only if there exists a continuous map $F : A \to B$ onto a metrizable space B such that for each $b \in B$, $F^{-1}(b)$ is metrizable. Tkachuk has provided some interesting and very useful inner characterizations of metrizably fibered spaces:

Theorem (Tkachuk, [14], Proposition 2.1)**.** *The following are equivalent for space* (X, T)*:*

- i. *X admits a continuous map p onto a second countable space M such that $p^{-1}(y)$ is metrizable for all $y \in M$,*
- ii. *X has a countable family γ of cozero open sets such that $\cup\gamma = X$ and the set $\gamma(x) = \cap\{U \in \gamma : x \in U\}$ is metrizable for any $x \in X$,*
- iii. *X has a family $\gamma \subset T - \{\emptyset\}$ as in ii. with the additional property that it is closed with respect to finite intersections and for any $x \in X$ and $U \in \gamma$ with $U \supset \gamma(x)$ there exists some $V \in \gamma$ such that $\gamma(x) \subset V \subset Cl(V) \subset U$, and*
- iv. *X has a countable family γ of zero sets such that $\cup\gamma = X$ and the set $\gamma(x) = \cap\{U \in \gamma : x \in U\}$ is metrizable for any $x \in X$.*

Harold Bennett and David Lutzer [1] have provided a related characterization in the class of GO-spaces (which we state here in the setting of compacta).

Theorem (Bennett and Lutzer, [1], Theorem 2.4)**.** *The following are equivalent for space* (X, T)*:*

- i. *X is a metrizably fibered compact ordered space;*
- ii. *X is compact and there exists a sequence $\{H(n) : n \geq 1\}$ of open covers of X such that for each $x \in X$, $\cap\{Star(x, H(n) : n \geq 1\}$ is a metrizable subspace of X;*

iii. *X admits a partition C into closed convex metrizable subspaces such that X/C is metrizable; and*
iv. *there is a continuous order-preserving map $G : X \to Y$ such that Y is a compact metrizable ordered space and $G^{-1}(y)$ is metrizable for each $y \in Y$.*

Recall that the lexicographic square is $S = [0,1] \times [0,1]$ with the order $<$ defined by $(x, y) < (x', y')$ if and only if $x < x'$ in $[0,1]$ or $x = x'$ and $y < y'$ in $[0,1]$, and that the double arrow space D is $([0,1] \times \{0,1\}) - \{(0,0),(1,1)\}$ with the relative topology of the lexicographic square S. In response to questions stated by Tkachuk [14, Question 3.5 and Question 3.6]), we also have the following:

Theorem (Daniel and Kennaugh, [3], Theorem 4)**.** *Every first countable continuous Hausdorff image of the lexicographic square is metrizably fibered.*

Therefore, a metrizably fibered compactum need not be perfectly normal in order for its first countable continuous images to be metrizably fibered. First countability is essential in the result above. Tkachuk [14, Example 2.3] has shown that every metrizably fibered compact space must be first countable, and Sibe Mardešić and Pavle Papić [8, Theorem 4] have constructed a non-first countable and therefore non-metrizably fibered continuous image of the lexicographic square.

Criteria for metrizability of metrizably fibered spaces have also been studied. Gary Gruenhage [6] has shown that every compact metrizably fibered space with a small diagonal is metrizable. (A space X has a *small diagonal* if for every uncountable subset Y of $X^2 - \Delta$ there is an open set $U \supset \Delta$ such that $Y - U$ is uncountable.) Additionally, metrizability of metrizably fibered spaces may be inferred from theorems of R. Engelking and A. Lelek in their study of weight of inverses.

Theorem (Engelking and Lelek, [4], Corollary 1.3)**.** *Let $f : X \to Y$ be a light open mapping of a compact space onto a locally connected infinite compactum Y. If there exists a dense subset A of Y such that $f^{-1}(y)$ is metrizable for each $y \in A$ then the weights of X and Y are equal.*

We establish some additional definitions and notation that we will use. A *continuum* is a compact connected Hausdorff space.

A Hausdorff space X is said to be an *IOK* if it is the continuous image of some compact ordered space. A Hausdorff space X is said to be an *IOC* if it is the continuous image of some ordered continuum (or *arc*). If K is an ordered space and $x, y \in K$ such that $x < y$, then the set $\{x, y\}$ is referred to as a *gap* of K if and only if $\{k \in K : x < k < y\}$ is empty.

3. Main Results

Theorem 3.1. *If X is a separable IOK then X is metrizably fibered.*

Proof: It follows from J. Nikiel, S. Purisch, and L. B. Treybig [10, Note 3.2] that X is the continuous image of the double arrow space D. D has projection onto first coordinate as a natural metrizable fibering and it is well-known that D is perfectly normal. Tkachuk [14, Corollary 2.6] has shown that the continuous image of a perfectly normal metrizably fibered space is metrizably fibered. □

Theorem 3.2. *Let L be a metrizably fibered compact ordered space. Then L has only countably many non-degenerate non-metrizable components.*

Proof: As in [1], let $f : L \to M$ be an onto continuous order-preserving map, M be a closed subspace of $[0, 1]$, and f have metric fibers.

Assume that there exists an uncountable collection $U = \{C_\alpha : C_\alpha = [a_\alpha, b_\alpha]$ is a non-degenerate component of L and C_α is non-metrizable$\}$. Since M has only countably many non-degenerate components then there exists a component C of M and an uncountable collection $U' = \{C_{\alpha'} : C_{\alpha'} = [a_{\alpha'}, b_{\alpha'}]$ is a non-degenerate component of L and $C_{\alpha'}$ is non-metrizable$\} \subseteq U$ such that $f(\cup U') \subset C$. Note that for each $C_{\alpha'} = [a_{\alpha'}, b_{\alpha'}] \in U'$ there exists $c_{\alpha'}$ and $d_{\alpha'}$ in $[a_{\alpha'}, b_{\alpha'}]$ so that $f(c_{\alpha'}) \neq f(d_{\alpha'})$ and $a_{\alpha'} \leq c_{\alpha'} < d_{\alpha'} \leq b_{\alpha'}$. Let D denote the usual metric on $M \subseteq [0, 1]$.

For each $C_{\alpha'} = [a_{\alpha'}, b_{\alpha'}] \in U'$, select $c_{\alpha'}$ and $d_{\alpha'}$ as above. Then there exists an $\epsilon > 0$ and an uncountable collection C' of such pairs of points such that $D(f(c), f(d)) \geq \epsilon$ for each pair $(c, d) \in C'$. Then there exist infinite subsequences $\{c_1, c_2, c_3, \dots\}$ and $\{d_1, d_2, d_3, \dots\}$ of elements of L so that $a_i \leq c_i \leq d_i \leq b_i$, $[a_i, b_i] \in U'$, and either $c_1 < d_1 \leq c_2 < d_2 \leq c_3 < d_3 \leq \cdots$ or $c_1 > d_1 \geq c_2 > d_2 \geq c_3 >$

$d_3 \geq \cdots$. In either case, the sequence is monotone so that there exists a $t \in L$ such that $c_i \to t$ and $d_i \to t$, where $t \in L$.

Consider the open ball $B = B(f(t)), \frac{\epsilon}{4})$ in C; then $f^{-1}(B)$ contains an open neighborhood of t in $f^{-1}(\cup U')$. Therefore, there exists $[a', b'] \in U'$ such that $[a', b'] \subseteq f^{-1}(B)$. With $(c', d') \in C'$ determined by $[a', b']$, we then have

$$\begin{aligned} \epsilon &\leq D(f(c'), f(d')) \\ &\leq D(f((c')), f(t)) + D(f(d'), f((t))) \\ &< \frac{\epsilon}{4} + \frac{\epsilon}{4} = \frac{\epsilon}{2}. \end{aligned}$$

This is clearly a contradiction so that L has only countably many non-degenerate non-metrizable components. □

Corollary 3.3. *If X is the continuous image of a compact ordered metrizably fibered space, then X has only countably many non-degenerate non-metrizable components.*

Theorem 3.4. *If X is the continuous Hausdorff image of a metrizably fibered compact ordered space, then X may be embedded in a continuous image of a metrizably fibered ordered continuum.*

Proof: Mardešić [7, Lemma 8]) has shown that each IOK X may be embedded in a natural way in an IOC Y. In particular, let $f : K \to X$ denote a continuous irreducible map of compact ordered space K onto X such that $f(a) \neq f(b)$ for each gap $\{a, b\}$ of K (see [15, Lemma 4]). Insert into each gap $\{a, b\}$ of K a copy $(0, 1)_{\{a,b\}}$ of the open interval $(0, 1)$ such that $(0, 1)_{\{a,b\}} \cap (0, 1)_{\{c,d\}} \neq \emptyset$ if and only if $\{a, b\} = \{c, d\}$.

Define $C = K \cup (\cup(0, 1)_{\{a,b\}})$, where the union ranges over all gaps $\{a, b\}$ of K, and the order $<_C$ is defined on C as follows: Let $<_K$ be the order on K and let x and y be distinct elements of C. Then

$x <_C y$ if x and y are elements of K and $x <_K y$, or
$x <_C y$ if x and y are elements of $(0, 1)_{\{a,b\}}$ for some gap $\{a, b\}$ of K and $x < y$ in $(0, 1)$, or
$x <_C y$ if $x \in K$ and $y \in (0, 1)_{\{a,b\}}$ for some gap $\{a, b\}$ of K and $x <_K a$ in K or $x = a$.

Then C is an ordered continuum. Consider X as being embedded in some cube $[0, 1]^k$. Then $f : K \to [0, 1]^k$ may be extended to a continuous map $f' : C \to [0,1]^k$ such that $f'|_K = f$ and $X \subseteq Y = f'(C)$.

We finally show that C in the above construction is metrizably fibered. Let $g : K \to N \subseteq [0,1]$ denote a continuous map of K into $[0,1]$ such that g has metric point inverses. In a manner parallel to the proof of Theorem 3.2 (and using the order-preserving property of g and the fact that $[0,1]$ contains only countably many mutually exclusive open subsets), there exist only countably many gaps $\{a,b\}$ of K such that $g(a) \neq g(b)$. For each gap $\{a,b\}$ of K, let $G_{\{a,b\}}$ denote $\{a,b\} \cup (0,1)_{\{a,b\}} \subseteq C$, and let $G_1 = \{\{a,b\} : \{a,b\}$ is a gap of K and $g(a) \neq g(b)\}$ and let $G_2 = \{\{a,b\} : \{a,b\}$ is a gap of K and $g(a) = g(b)\}$. From above, G_1 is countable. For each $\{a,b\} \in G_1$, let $g_{\{a,b\}}$ denote a linear map of $G_{\{a,b\}}$ onto $\{g(a), g(b)\} \cup (0,1)_{\{g(a),g(b)\}} \subseteq C$ such that $g_{\{a,b\}}(a) = g(a)$ and $g_{\{a,b\}}(b) = g(b)$.

Define $G : C \to [0,1]$ by

$$G(x) = \begin{cases} g(x), \ x \in K \text{ but } x \text{ is the endpoint of no gap in } K, \\ g(a), \ x \in G_{\{a,b\}} \text{ for some } \{a,b\} \in G_2, \\ g_{\{a,b\}}(x), \ x \in G_{\{a,b\}} \text{ and } \{a,b\} \in G_1. \end{cases}$$

The proof that G is continuous and has metric point inverses is straightforward and is left to the reader. □

B. J. Pearson [11] and Joseph N. Simone [12] each investigate the "metric components" of a Hausdorff space. In [12], for example, X is a Suslinian IOK, and a relation R on X is defined by xRy provided there exists a metrizable continuum in X containing x and y. For each $x \in X$, define $M_x = \{y \in X : xRy\}$. M_x is called the metric component of x. Simone further shows that R is an equivalence relation, and if X is also first countable, then M_x is a continuum for each $x \in X$.

The following two results follow closely related results from [2] and [3]. We include them here so that this treatment is self-contained.

Theorem 3.5. *Let X denote a first countable IOC. Then there exists a continuous mapping $p : X \to M$ from X onto a metrizable space M such that for all $m \in M$, $p^{-1}(m)$ is a continuum in X and for all distinct $a, b \in p^{-1}(m)$, there exists a metrizable continuum in $p^{-1}(m)$ containing a and b.*

Proof: Let $f : K \to X$ denote a continuous map of an arc K onto X where K is metrizably fibered. As in [1], there is a continuous order-preserving map $g : K \to [0,1]$ with metric fibers.

For each $x \in X$, define $M_x = \{y \in X :$ there exists a separable subcontinuum of X containing x and $y\}$. By [16], $M_x = \{y \in X :$ there exists a metrizable subcontinuum of X containing x and $y\}$. Let $G = \{M_x : x \in X\}$.

We first show that M_x is a continuum for each $x \in X$. M_x is clearly connected so we show that it is closed. Let p be a limit point of M_x and let $\{B_n\}$ be a countable basis at p. For each n, let $x_n \in (B_n \cap M_x)$. For each n, select a separable subcontinuum M_n in X such that M_n contains x_n and x. Then $\mathrm{Cl}(\cup M_n)$ is a separable continuum in X containing both x and p. Therefore, $p \in M_x$ and M_x is closed.

Assume that G is not upper semi-continuous. Then there exists $g = M_x \in G$, $z \in g$, a countable basis $\{B_n\}$ at z, an open set U such that $g \subset U$, and a sequence $\{h_n\}$ of mutually disjoint separable subcontinua of X such that

i) $h_n \cap g = \emptyset$ for all n,
ii) $h_n \cap B_n \neq \emptyset$ for each n, and
iii) $h_n \cap (X - U) \neq \emptyset$ for each n.

For each n, select $z_n \in (B_n \cap h_n)$ and $u_n \in (h_n \cap \mathrm{Bd}(U))$. Then $z_n \to z$ and $u_n \to u$ for some $u \in \mathrm{Bd}(U)$. It then follows from Theorem 4.1 of [2] that the limiting set h of $\{h_n\}$ is a metrizable continuum containing z and u. Therefore, $u \in g$ which is a contradiction.

G is then an upper semi-continuous decomposition of X into continua. Let $p : X \to X/G = M$ be the natural map associated with the decomposition space X/G.

Let Q denote the set of rational numbers in $[0,1]$. For each $q \in Q$, let D_q denote a countable dense subset of $g^{-1}(q)$. Consider $p \circ f(\cup_{r\in Q} D_r) \subseteq X/G$, and let $O = X/G - \mathrm{Cl}_{X/G}[p \circ f(\cup_{r\in Q} D_r)] \subseteq X/G$. Since $g : K \to [0,1]$ is order-preserving, it follows that $g(f^{-1} \circ p^{-1}(O))$ is degenerate, say $g(f^{-1} \circ p^{-1}(O)) = n$. Then $g^{-1}(n) = f^{-1} \circ p^{-1}(O)$ is separable metric. It then follows that either K is separable metric (by [16]) or that O is empty and $p \circ f(\cup_{r\in Q} D_r)$ is therefore countable dense in X/G.

X/G is therefore a separable IOC and is metrizable by [16]. $\square$

$$\begin{array}{ccc} & f & \\ K & \longrightarrow & X \\ g \downarrow & & \downarrow p \\ [0,1] & & M = X/G \end{array}$$

As in the diagram above, we will henceforth assume that the mappings g, f, and p, the collection G, and the spaces K, X, $M = X/G$, and M_x will be as in Theorem 3.5.

For fixed but arbitrary $x \in X$, consider $M_x \in G$. Let $\mathcal{L}$ be the set of all collections $\mathcal{M} = \{m_\alpha\}$ of disjoint non-degenerate metrizable continua in M_x such that if $\mathcal{M} \in \mathcal{L}$, $m \in \mathcal{M}$, and if $f(J) \cap m \neq \emptyset$ where J is a non-degenerate metric arc in K, then $f(J) \subseteq m$. Since any chain in $\mathcal{L}$ has its union as an upper bound, $\mathcal{L}$ has a maximal element $\mathcal{N}$.

Theorem 3.6. *For each $x \in X$, M_x is metrizable.*

Proof: Select and fix $x \in X$, and consider M_x. Let $\mathcal{N}'$ denote the subset of $\mathcal{N}$ such that $\mathcal{N}' = \{n \in \mathcal{N} : g(f^{-1}(n))$ is non-degenerate$\}$. Since $[0,1]$ does not contain uncountably many mutually exclusive open subintervals, it follows (since g is order-preserving) that $\mathcal{N}'$ is countable. Then by definition of M_x and the theorem of Treybig [16], there exists a metrizable continuum C' in M_x containing $\cup\mathcal{N}'$. If $\mathcal{N}' = \emptyset$, select any element of $\mathcal{N}$ to be C'.

Let $S' = \{s : s$ is a non-degenerate closed subinterval of K and $f(s)$ is a degenerate element of $M_x\}$. Let $E' = \{e'_1, e'_2, \dots\}$ denote a countable dense subset of $g(\cup S')$. For each $e'_j \in E'$, select a countable dense subset D_j of $g^{-1}(e'_j)$ and define $D = \cup D_j$. We will show that $f(D)$ is dense in $\mathrm{Cl}(f(\cup S'))$. Let O be open in $\mathrm{Cl}(f(\cup S'))$ and let $y \in f(\cup S') \cap O$. Suppose $s_y \in S'$, $s_y = [l_y, r_y] \subseteq K$, and $f(s_y) = y$. We may clearly assume that s_y is maximal in the sense that if $t_y \in S'$ and $f(t_y) = y$, then $t_y \subseteq s_y$.

We now consider three possibilities. If $g(s_y)$ is a non-degenerate closed subinterval of $[0,1]$, then $g(s_y) \cap E' \neq \emptyset$. Select $e'_k \in E'$ and $e \in D_k \cap s_y$. Then $f(e) = y \in O$. Now suppose that $g(s_y)$ is a degenerate element z_y of $[0,1]$ and z_y is isolated in $g(\cup S')$. Then $z_y \in E'$. Letting $z_y = e'_m \in E'$, there is $e \in D_m \cap s_y$. Then

$f(e) = y \in O$. As a final case, suppose that $g(s_y)$ is a degenerate element z_y of $[0,1]$ and z_y is not isolated in $g(\cup S')$. Let $\{B_n\}$ denote a countable local basis at z_y by connected open sets of $[0,1]$. For each n, let C_n denote the open component of $g^{-1}(B_n \cap g(\cup S'))$ that contains s_y. By the maximality of s_y, $\cap C_n = s_y$. For each n, select $l_n \in C_n \cap D$. We may without loss of generality assume that $\{l_n\}$ converges to l_y. Then $\{f(l_n)\}$ converges to y by first countability. The tail of the sequence $\{f(l_n)\}$ is therefore contained in O.

For each $d \in f(D)$, let Z_d denote a metrizable subcontinuum containing d and C'. Then $C = Cl(\cup\{Z_d : d \in f(D)\})$ is a separable connected IOK and therefore metrizable, and contains C' and $Cl(f(\cup S'))$. If $S' = \emptyset$, set $C = C'$.

Let S_2 denote an irreducible closed subset of K that maps onto M_x and $f(x) \neq f(y)$ for each gap $\{x,y\} \in S_2$. As in the proof of Theorem 3.4, each gap $\{x,y\}$ of S_2 is "filled" with a copy of the open interval $(0,1)$, and the resulting space S_1 is given the natural ordering. Then a continuous mapping $h : S_1 \to Y$ is defined so that Y is a locally connected continuum, $M_x \subseteq Y$, and $h|_{S_2} = f|_{S_2}$.

Then by Jacek Nikiel [9], there exists a closed metrizable set $T \subseteq M_x$ such that $C \subseteq T$ and each component of $Y - T$ has a two-point (or fewer) boundary. Moreover, those components with two-point boundaries form a countable family since T is metrizable, while those components with one-point boundaries form a countable family since M_x and therefore X would otherwise fail to be first countable. Each component of $Y - T$ is contained in some element of $\mathcal{N} - \mathcal{N}'$. Each such component is separable, and therefore, M_x is separable and metrizable by [16]. □

We have therefore shown the following.

Theorem 3.7. *Every first countable Hausdorff space X that is the continuous image of a metrizably fibered ordered continuum is metrizably fibered.*

Proof: $p : X \to X/G$ is a continuous map from X onto the compact metric space X/G. Each element of X/G has compact metric fiber M_x for some $x \in X$. □

Corollary 3.8. *Every first countable Hausdorff space X that is the continuous image of a metrizably fibered compact ordered space is metrizably fibered.*

Proof: By Theorem 3.4, there exists a metrizably fibered ordered continuum L and a continuous mapping $F : L \to Y$ such that X is embedded as a closed subset of Y. Then by Theorem 3.7, there exists a continuous map $p_Y : Y \to N$ such that N is compact metric and p_Y has metric point inverses. The restriction $p = p_Y|_X$ is then the required mapping. □

The general question [14, Problem 3.3] whether each first countable continuous image of a metrizably fibered compact space is metrizably fibered remains open.

Acknowledgment. The author wishes to express his gratitude to the referee for a number of very helpful suggestions and comments.

References

[1] Harold Bennett and David Lutzer, *Metrizably fibered generalized ordered spaces*, Topology Appl. **130** (2003), no. 3, 283–290.

[2] D. Daniel and C. T. Kennaugh, *Concerning metrizable continua of convergence*, Topology Proc. **27** (2003), no. 1, 101–109.

[3] ______, *Metrizable fiberings of continuous images of the lexicographic square*, Topology Appl. **153** (2006), no. 10, 1603–1608.

[4] R. Engelking and A. Lelek, *Metrizability and weight of inverses under confluent mappings*, Colloq. Math. **21** (1970), 239–246.

[5] J. Gerlits and Z. Szentmiklóssy, *On compact fibered spaces*, Topology Appl. **123** (2002), no. 1, 87–90.

[6] Gary Gruenhage, *Spaces having a small diagonal*, Topology Appl. **122** (2002), no. 1-2, 183–200.

[7] Sibe Mardešić, *Images of ordered compacta are locally peripherally metric*, Pacific J. Math. **23** (1967), 557–568.

[8] Sibe Mardešić and Pavle Papić, *Continuous images of ordered compacta, the Suslin property and diadic compacta*, Glasnik Mat.-Fiz. Astronom. Drukštvo Mat. Fiz. Hrvatske SER. II **17** (1962), 3–25 (1963).

[9] Jacek Nikiel, *Images of arcs—a nonseparable version of the Hahn-Mazurkiewicz theorem*, Fund. Math. **129** (1988), no. 2, 91–120.

[10] J. Nikiel, S. Purisch, and L. B. Treybig, *Separable zero-dimensional spaces which are continuous images of ordered compacta*, Houston J. Math. **24** (1998), no. 1, 45–56.

[11] B. J. Pearson, *A Hahn-Mazurkiewicz theorem for hereditarily locally connected continua*, Glas. Mat. Ser. III **19(39)** (1984), no. 2, 323–334

[12] Joseph N. Simone, *Metric components of continuous images of ordered compacta*, Pacific J. Math. **69** (1977), no. 1, 269–274.

[13] Michael G. Tkačenko, *$\mathcal{P}$-approximable compact spaces*, Comment. Math. Univ. Carolin. **32** (1991), no. 3, 583–595.

[14] Vladimir V. Tkachuk, *A glance at compact spaces which map "nicely" onto the metrizable ones*, Topology Proc. **19** (1994), 321–334.

[15] L. B. Treybig, *Concerning continuous images of compact ordered spaces*, Proc. Amer. Math. Soc. **15** (1964), 866–871.

[16] ———, *Concerning continua which are continuous images of compact ordered spaces*, Duke Math. J. **32** (1965), 417–422.

Department of Mathematics; Lamar University; Beaumont, Texas 77710

E-mail address: dale.daniel@lamar.edu