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# $\beta X \setminus \{p\}$ ARE NON-NORMAL FOR NON-DISCRETE SPACES $X$

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Abstract. For a non-compact metrizable space $X$ without isolated points and a point $p \in \beta X \setminus X$, the subspace $\beta X \setminus \{p\}$ is shown to be non-normal.

The proof is based on a special $\pi$-base of $X$, which the author devised earlier out of Arhangel'skii's regular base. Maximal disjoint families of basic open sets are introduced, and ultrafilters on them play a prominent role.

## 1. Introduction

This is both a sequel to and a revision of the author's earlier paper [5].

It is an interesting but difficult question whether $\beta\omega \setminus \{p\}$ is normal or not for a point $p \in \beta\omega \setminus \omega$ (see, e.g., [2]). Under CH, it is known to be non-normal, and a few proofs are offered. But, without CH or under the negation of CH, we seem to know very little.

On the other hand, if we were able to prove the non-normality of $\beta\omega \setminus \{p\}$, then it would immediately imply the non-normality of $\beta X \setminus \{p\}$ for any non-pseudocompact space $X$ and some $p \in \beta X \setminus X$ because $\omega$ is $C$-embedded in $X$. This observation motivated our

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earlier study [5] of $\beta X \setminus \{p\}$ for a non-compact metrizable space $X$ without isolated points. Here we extend the argument and show within ZFC the following theorem.

**Theorem.** *If $X$ is a non-compact, metrizable space without isolated points and $p \in \beta X \setminus X$ is an arbitrary point, then $\beta X \setminus \{p\}$ is not normal.*

In [5], this theorem was shown under the additional assumption that either $X$ is strongly 0-dimensional or $p$ is a remote point. For further background of this topic, as well as terminologies, the readers are referred to [1], [2], [3], [4], [5].

In particular, a point $x \in \Omega$ is called a *butterfly point* of a Tychonoff space $\Omega$ if there are closed sets $F_0$, $F_1$ of $\Omega$ such that (1) $x \in \mathrm{Cl}_\Omega (F_i \setminus \{x\})$ for each $i$, and (2) $F_0 \cap F_1 = \{x\}$. This is due to B. È. Šapirovskiĭ [4]. As we noted in [5] for the proofs of Theorem 1 and Theorem 2, it is necessary and sufficient to show that $p$ is a butterfly point of $\beta X$ in our theorem above (sufficient because $X \subset \beta X \setminus \{p\}$ is $C^*$-embedded in $\beta X$).

Throughout the paper, for a family $\mathcal{A}$ of subsets of $X$, we will use the symbol

$$\mathcal{A}^* = \bigcup \{S : S \in \mathcal{A}\}.$$

A family $\mathcal{A}$ of nonempty open sets of $X$ is called *a maximal disjoint family* if it is disjoint and is not a proper subfamily of any disjoint family of nonempty open sets. Below we will often say "such $\mathcal{A}$ covers $X$ modulo nowhere dense set." This makes sense because, although $\mathcal{A}$ does not necessarily cover $X$, the set $X \setminus \mathcal{A}^*$ can contain no nonempty open set of $X$. In [5], we called such family $\mathcal{A}$ a "dense cover."

## 2. From the previous construction

We begin with the fundamental construction from [5].

The following is shown in [5, pp. 340–342], using Arhangel'skii's regular base.

**Proposition 1.** *Let $X$ be a metrizable space without isolated points. Then $X$ has a $\pi$-base $\mathcal{B}$ consisting of nonempty open sets, each $B$ being associated with three sets $B^{(i)} \in \mathcal{B}$, $i = 0, 1, 2$, such that*

(1) $B \supset \mathrm{Cl}_X B^{(i)}$ *and* $\mathrm{Cl}_X B^{(i)} \cap \mathrm{Cl}_X B^{(j)} = \emptyset$ *for* $i \neq j$*;*

(2) *if* $B \subsetneq C$ *then either* $B \subset C^{(i)}$ *for some* $i$ *or* $B \cap C^{(i)} = \emptyset$ *for any* $i$*;*
(3) *for each* $B, C$*, either* $B \cap C = \emptyset$*,* $B \subset C$*, or* $B \supset C$*;*
(4) *each* $B$ *is contained in only finitely many members of* $\mathcal{B}$*;*
(5) *every open cover of* $X$ *is refined by a locally finite (in* $X$*), maximal disjoint subfamily of* $\mathcal{B}$*.*

Note that (1) corresponds to (1′) and (5) is shown in [5, p. 342, lines 11–18].

In Proposition 1, let $\mathcal{B}_0$ be the set of maximal members of $\mathcal{B}$, and, for $n \geq 1$, let $\mathcal{B}_n$ be the set of maximal members of $\mathcal{B} \setminus \bigcup_{m<n} \mathcal{B}_m$ ("maximal" in the sense of set-inclusion). Then it follows from (3) and (4) that $\mathcal{B} = \bigcup_n \mathcal{B}_n$. This leads us to the following equivalent formulation of Proposition 1.

**Proposition 2** (Alternate)**.** *Let* $X$ *be a metrizable space without isolated points. Then* $X$ *has a* $\pi$*-base*

$$\mathcal{B} = \bigcup_{n=0}^{\infty} \mathcal{B}_n$$

*such that*

- *each* $\mathcal{B}_n$ *is a locally finite (in* $X$*), maximal disjoint family of nonempty open sets;*
- $\mathcal{B}_{n+1}$ *refines* $\mathcal{B}_n$*;*
- *for each* $B \in \mathcal{B}_n$*, there are three sets* $B^{(i)} \in \mathcal{B}_{n+1}$*,* $i = 0, 1, 2$*, such that* $\mathrm{Cl}_X B^{(i)} \subset B$ *and* $\mathrm{Cl}_X B^{(i)} \cap \mathrm{Cl}_X B^{(j)} = \emptyset$ *for* $i \neq j$*;*
- *every open cover of* $X$ *is refined by a locally finite (in* $X$*), maximal disjoint subfamily of* $\mathcal{B}$*.*

Throughout the rest of the paper, we will refer to Proposition 1; Proposition 2 is included here just for the readers' convenience.

## 3. Locally finite, maximal disjoint families

From this point onward, we fix a non-compact metrizable space $X$ without isolated points, any point $p \in \beta X \setminus X$, and the family $\mathcal{B}$ as in Proposition 1, and let $\Xi$ denote the family of all locally finite (in $X$), maximal disjoint subfamilies of $\mathcal{B}$ of the space $X$. First of all, we suppose that $\Xi$ is well-ordered in an arbitrary way.

Now let us take by recursion $\xi_\lambda \in \varXi$ and an ultrafilter $\varphi_\lambda$ on $\xi_\lambda$ ($\varphi_\lambda$ consists of subfamilies of $\xi_\lambda$), for ordinals $\lambda < \theta$, that satisfy the following:

(a) $p \in \mathrm{Cl}_{\beta X}\mathcal{U}^*$ for any $\mathcal{U} \in \varphi_\lambda$;
(b) for $\lambda < \mu$, there is a $\mathcal{U} \in \varphi_\lambda$ such that $U \cap V \neq \emptyset$, $U \in \mathcal{U}$ and $V \in \xi_\mu$ imply $V \subset U$;
(c) for $\lambda < \mu$ and $\mathcal{U} \in \varphi_\lambda$, the set $\{V \in \xi_\mu : V \subset U$ for some $U \in \mathcal{U}\}$ belongs to $\varphi_\mu$;
(d) for $\xi \in \varXi \setminus \{\xi_\lambda : \lambda < \theta\}$, there is a $\lambda < \theta$ such that none of $\mathcal{U} \in \varphi_\lambda$ satisfies (b) with $\xi_\mu$ replaced by $\xi$.

For $\lambda < \theta$ and a neighborhood $O$ of $p$ in $\beta X$, let us define

$$\xi_\lambda(O) = \{U \in \xi_\lambda : U \cap O \neq \emptyset\}.$$

Then condition (a) is equivalent to

$$\varphi_\lambda \ni \xi_\lambda(O) \text{ for every } O.$$

For $\mathcal{U}$ satisfying (b), we will say, "$\mathcal{U}$ is partitioned by $\xi_\mu$." This makes sense because, since $\xi_\mu$ is a maximal disjoint family, (b) means that each $U \in \mathcal{U}$ is expressed as the union of members of $\xi_\mu$ *modulo nowhere dense set*.

Condition (b) also implies that $\varphi_\lambda$ has a filter base, each member of which is partitioned by $\xi_\mu$, because $\mathcal{S} \cap \mathcal{U} \in \varphi_\lambda$ for every $\mathcal{S} \in \varphi_\lambda$, and $\mathcal{S} \cap \mathcal{U}$ is also partitioned by $\xi_\mu$.

For each $\mathcal{U} \in \varphi_\lambda$ and $\lambda < \mu$, let us define

$$\xi_\mu(\mathcal{U}) = \{V \in \xi_\mu : V \subset U \text{ for some } U \in \mathcal{U}\}.$$

Then condition (c) is stated more briefly:

For $\lambda < \mu$ and $\mathcal{U} \in \varphi_\lambda$, the set $\xi_\mu(\mathcal{U})$ belongs to $\varphi_\mu$.

Condition (d) means that the family $\{\xi_\lambda : \lambda < \theta\}$ is maximal in the sense of (b) in $\varXi$.

For the first step of the recursion, let $\xi_0$ be the first member of $\varXi$ and $\varphi_0$ be any ultrafilter that satisfies condition (a).

Now suppose that, for all ordinals $\lambda < \mu$, we have $\xi_\lambda$ and $\varphi_\lambda$ that satisfy the above conditions.

If there is no $\xi \in \varXi$ such that

$$\text{for any } \lambda < \mu, \text{ some } \mathcal{U} \in \varphi_\lambda \text{ is partitioned by } \xi, \tag{6}$$

then we terminate the recursion and define $\theta = \mu$. Obviously, by (1), such $\theta$ is not an isolated ordinal. Otherwise, let $\xi_\mu$ be the first $\xi \in \Xi$ that satisfies (6).

Before defining $\varphi_\mu$ that satisfies (c) and (a), we need to check that

(i) the sets $\xi_\mu(\mathcal{U})$ satisfy the finite intersection property,
(ii) $\xi_\mu(\mathcal{U}) \cap \xi_\mu(O) \neq \emptyset$ for any neighborhood $O$ of $p$.

For (i), take $\lambda_0 \leq \lambda_1 \leq \cdots \leq \lambda_n < \mu$ and $\mathcal{U}_i \in \varphi_{\lambda_i}$. Let us show

$$\bigcap_{i \leq n} \xi_\mu(\mathcal{U}_i) \neq \emptyset.$$

Considering the filter base of $\varphi_\lambda$, we may suppose that each of $\mathcal{U}_0, \mathcal{U}_1, \ldots, \mathcal{U}_n$ is partitioned by $\xi_\mu$. Since $\mathcal{U}_n \in \varphi_{\lambda_n}$ and $\xi_{\lambda_n}(\mathcal{U}_i) \in \varphi_{\lambda_n}$ for $i < n$, by the recursive assumption, we have

$$\mathcal{U}_n \cap \bigcap_{i < n} \xi_{\lambda_n}(\mathcal{U}_i) \neq \emptyset.$$

Take $V$ from this intersection arbitrarily. Since $\xi_\mu$ covers $X$ modulo nowhere dense set, $S \cap V \neq \emptyset$ for some $S \in \xi_\mu$.

Then $V \in \mathcal{U}_n$ implies $S \subset V$ and $S \in \xi_\mu(\mathcal{U}_n)$. Furthermore, $V \in \xi_{\lambda_n}(\mathcal{U}_i)$, $i < n$, implies $V \subset U$ for some $U \in \mathcal{U}_i$. Hence, $S \subset U$, and we finally have

$$S \in \xi_\mu(\mathcal{U}_i) \text{ for all } i \leq n.$$

To see (ii), again it suffices to consider the case that $\mathcal{U}$ is partitioned by $\xi_\mu$. Take any neighborhood $O$ of $p$ in $\beta X$, and let

$$\mathcal{S} = \mathcal{U} \cap \xi_\lambda(O).$$

By (a), we have $\mathcal{S} \in \varphi_\lambda$. Take any $S \in \mathcal{S}$. Then $S$ meets $O$ and is covered by members of $\xi_\mu$ modulo nowhere dense set. Hence, $W \cap S \cap O \neq \emptyset$ for some $W \in \xi_\mu(\mathcal{S})$. This shows

$$W \in \xi_\mu(\mathcal{S}) \cap \xi_\mu(O) \subset \xi_\mu(\mathcal{U}) \cap \xi_\mu(O) \neq \emptyset.$$

Now we can take an ultrafilter $\varphi_\mu$ on the set $\xi_\mu$ that contains all $\xi_\mu(O)$ and all $\xi_\mu(\mathcal{U})$, and the construction is complete.

## 4. Lemmas

For each $\lambda < \theta$, let us define

$$H_\lambda = \bigcap\{\mathrm{Cl}_{\beta X}\mathcal{U}^* : \mathcal{U} \in \varphi_\lambda\}.$$

Then obviously, $p \in H_\lambda$.

**Lemma 1.** *For $\lambda < \mu$, $H_\lambda \supset H_\mu$.*

*Proof:* Take any $\mathcal{U} \in \varphi_\lambda$. Then $\xi_\mu(\mathcal{U})^* \subset \mathcal{U}^*$ and $\xi_\mu(\mathcal{U}) \in \varphi_\mu$ by (c). Thus, we have

$$H_\mu = \bigcap\{\mathrm{Cl}_{\beta X}\mathcal{V}^* : \mathcal{V} \in \varphi_\mu\} \subset \mathrm{Cl}_{\beta X}(\xi_\mu(\mathcal{U})^*) \subset \mathrm{Cl}_{\beta X}\mathcal{U}^*$$

and hence, $H_\mu \subset H_\lambda$. □

**Lemma 2.** *For any neighborhood $O$ of $p$ in $\beta X$, there is a $\lambda < \theta$ such that $H_\lambda \subset O$.*

*Proof:* First, let us take neighborhoods $P$, $Q$ of the point $p$ such that

$$p \in Q \subset \mathrm{Cl}_{\beta X}Q \subset P \subset \mathrm{Cl}_{\beta X}P \subset O.$$

For each point $x \in X$, we can take an open neighborhood $N(x)$ of $x$ in $X$ such that $N(x) \subset P$ or $N(x) \cap \mathrm{Cl}_X(Q \cap X) = \emptyset$, depending on whether $x$ belongs to $\mathrm{Cl}_X(Q \cap X)$ or not. By (5) of Proposition 1, the cover $\{N(x) : x \in X\}$ is refined by a family $\xi \in \Xi$. Obviously, we have $\xi(Q)^* = \{U \in \xi : U \cap Q \neq \emptyset\}^* \subset P$.

If $\xi = \xi_\lambda$ for some $\lambda < \theta$, then, by (a), $\xi(Q) \in \varphi_\lambda$. Then we have

$$H_\lambda \subset \mathrm{Cl}_{\beta X}\xi(Q)^* \subset \mathrm{Cl}_{\beta X}P \subset O.$$

If $\xi \neq \xi_\lambda$ for any $\lambda < \theta$, then, by (d), there is a $\lambda$ such that none of $\mathcal{U} \in \varphi_\lambda$ is partitioned by $\xi$.

Now, for every $\mathcal{U} \in \varphi_\lambda$, consider $\mathcal{U} \cap \xi_\lambda(Q) \in \varphi_\lambda$. Then there are $U = U_\mathcal{U} \in \mathcal{U} \cap \xi_\lambda(Q)$ and $V = V_\mathcal{U} \in \xi$ such that $U \cap V \neq \emptyset$ and $V \not\subset U$. By (3) of Proposition 1, in this case $V \supset U$ holds.

Let $\mathcal{S} = \{U_\mathcal{U} : \mathcal{U} \in \varphi_\lambda\}$. Then we have $\mathcal{S} \in \varphi_\lambda$, because $\mathcal{S}$ meets every $\mathcal{U} \in \varphi_\lambda$ and $\varphi_\lambda$ is an ultrafilter. On the other hand, we have $U_\mathcal{U} \cap Q \neq \emptyset$, $V_\mathcal{U} \cap Q \neq \emptyset$, and hence, $V_\mathcal{U} \subset P$. All these imply

$$H_\lambda \subset \mathrm{Cl}_{\beta X}\mathcal{S}^* = \mathrm{Cl}_{\beta X}\Big(\bigcup_\mathcal{U} U_\mathcal{U}\Big) \subset \mathrm{Cl}_{\beta X}\Big(\bigcup_\mathcal{U} V_\mathcal{U}\Big) \subset \mathrm{Cl}_{\beta X}P \subset O. \quad \square$$

For the rest of our argument, the following symbol will be useful for $\lambda < \theta$ and $i = 0, 1, 2$:

$$\mathcal{L}_{\lambda,i} = \{U^{(i)} : U \in \xi_\lambda\}.$$

**Lemma 3.** *For any $\lambda < \theta$ and $i < 3$, there is a point $r_{\lambda,i} \in H_\lambda$ such that*

$$r_{\lambda,i} \in \mathrm{Cl}_{\beta X} \mathcal{L}^*_{\mu,i} \ \textit{whenever}\ \lambda < \mu.$$

*Proof:* To show this, it suffices to see that the family consisting of

$$\mathcal{U}^*,\ \mathcal{U} \in \varphi_\lambda,\ \text{ and }\ \mathcal{L}^*_{\mu,i},\ \ \mu > \lambda$$

has the finite intersection property; i.e., for $\mathcal{U}$ and $\mu_0, \dots, \mu_{n-1} > \lambda$,

$$\mathcal{U}^* \cap \bigcap_{j<n} \mathcal{L}^*_{\mu_j,i} \neq \emptyset.$$

(Note that it does not make the proof any easier if we invoke the linear relation, say, $\mu_0 \geq \cdots \geq \mu_{n-1} > \lambda$.)

As we have noted above, $\mathcal{U}$ contains a subfamily $\mathcal{U}_j \in \varphi_\lambda$, for each $j < n$, which is partitioned by $\xi_{\mu_j}$.

Take any $U_0 \in \mathcal{U} \cap \bigcap_{j<n} \mathcal{U}_j$.

If $U_0 \in \xi_{\mu_j}$ for all $j < n$, then we have nothing more to do because $U_0^{(i)} \subset \mathcal{U}^* \cap \bigcap_{j<n} \mathcal{L}^*_{\mu_j,i}$.

Otherwise, consider $J_1 = \{j : U_0 \notin \xi_{\mu_j}\}$. For each $j \in J_1$, since $U_0$ is covered by $\xi_{\mu_j}$ modulo nowhere dense set and $U_0^{(i)} \subset U_0$, some $S \in \xi_{\mu_j}$ satisfies $U_0^{(i)} \cap S \neq \emptyset$ and $S \subset U_0$. By (3) and (4) of Proposition 1, we can find a maximal one $U_1$ among such $S$'s, as $j$ runs over $J_1$ ("maximal" in the sense of set-inclusion). Then by (2), we have $U_1 \subset U_0^{(i)}$.

If $U_1 \in \xi_{\mu_j}$ for all $j \in J_1$, then again we have nothing more to do because $U_1^{(i)} \subset \mathcal{U}^* \cap \bigcap_{j<n} \mathcal{L}^*_{\mu_j,i}$. Otherwise, consider $J_2 = \{j \in J_1 : U_1 \notin \xi_{\mu_j}\}$. For each $j \in J_2$, since $U_0$ is covered by $\xi_{\mu_j}$ modulo nowhere dense set and $U_1^{(i)} \subset U_1 \subset U_0^{(i)} \subset U_0$, some $S \in \xi_{\mu_j}$ satisfies $U_1^{(i)} \cap S \neq \emptyset$. The maximality of $U_1$ on the previous stage implies $S \subset U_1$. By (3) and (4) of Proposition 1, we can find a maximal one $U_2$ among such $S$'s, as $j$ runs over $J_2$. Then by (2), we have $U_2 \subset U_1^{(i)}$.

Next consider $U_2$ and proceed similarly. This process eventually terminates because we have only finitely many $j$'s. This concludes the proof. $\square$

## 5. Proof of theorem

Now we are ready to prove the Theorem.

For each $i < 3$, let

$$K_i = \{r_{\lambda,i} : \lambda < \theta\}.$$

By lemmas 1, 2, and 3, for any neighborhood $O$ of $p$, there is $\lambda < \theta$ such that

$$r_{\mu,i} \in H_\mu \subset H_\lambda \subset O \text{ for all } \mu \geq \lambda,$$

and hence, $p \in \mathrm{Cl}_{\beta X} K_i$.

Take and fix $\lambda < \theta$ arbitrarily. Then

$$K_i = \{r_{\mu,i} : \mu \geq \lambda\} \cup \{r_{\mu,i} : \mu < \lambda\} \subset H_\lambda \cup \mathrm{Cl}_{\beta X} \mathcal{L}^*_{\lambda,i}\,.$$

On the other hand, by (1) of Proposition 1 and the local-finiteness of $\xi_\lambda$,

$$\mathrm{Cl}_X \mathcal{L}^*_{\lambda,i} \cap \mathrm{Cl}_X \mathcal{L}^*_{\lambda,j} = \emptyset \text{ for } i \neq j,$$

and hence, by the metrizability of $X$,

$$\mathrm{Cl}_{\beta X} \mathcal{L}^*_{\lambda,i} \cap \mathrm{Cl}_{\beta X} \mathcal{L}^*_{\lambda,j} = \emptyset \text{ for } i \neq j. \tag{7}$$

This implies

$$\mathrm{Cl}_{\beta X} K_i \cap \mathrm{Cl}_{\beta X} K_j \subset H_\lambda \text{ for any } \lambda \text{ and } i \neq j,$$

and, by Lemma 2,

$$\mathrm{Cl}_{\beta X} K_i \cap \mathrm{Cl}_{\beta X} K_j \subset \bigcap_\lambda H_\lambda = \{p\} \text{ for } i \neq j\,.$$

Thus, $p$ is a butterfly point of $\beta X$.

It might happen that, for some $i$, $r_{\lambda,i} = p$ for sufficiently large $\lambda$ and $p \notin \mathrm{Cl}_{\beta X}(K_i \setminus \{p\})$. But, by Lemma 3 and (7), there is at most one such $i$, so that it does not matter; in fact, this is the reason we have constructed three sequences $\{r_{\lambda,i} : \lambda\}$, $i < 3$.

## 6. Remarks

**Remark 1.** Note that our argument works for any non-compact normal space which has a $\pi$-base as in Proposition 1.

In particular, if a non-compact normal space $X$ contains a dense metrizable subspace without isolated points, then $\beta X \setminus \{p\}$ is non-normal for any $p \in \beta X \setminus X$.

**Remark 2.** In [5], we required

$$p \notin \mathrm{Cl}_{\beta X} B \text{ for all } B \in \mathcal{B}, \tag{8}$$

but not here. There is no particular reason for this; $\mathcal{B}$ is easily modified to satisfy (8).

On the other hand, if (8) is satisfied, condition (a) implies that each $\varphi_\lambda$ is free; i.e., $\bigcap \varphi_\lambda = \emptyset$. This further implies $H_\lambda \cap X = \emptyset$, because $\xi_\lambda$ is locally finite in $X$, and hence, $K_i \subset \beta X \setminus X$. Thus, we have nonempty closed sets $\mathrm{Cl}_{\beta X} K_i \setminus X$, $i < 3$, which guarantee that $p$ is a butterfly point of $\beta X \setminus X$. However, how it is related to the non-normality of $\beta X \setminus (X \cup \{p\})$ is a delicate question even if $X$ is locally compact, as we noted in [5].

## References

[1] Ryszard Engelking, *General Topology.* Translated from the Polish by the author. 2nd ed. Sigma Series in Pure Mathematics, 6. Berlin: Heldermann Verlag, 1989.

[2] Klaas Pieter Hart and Jan van Mill, *Open problems on $\beta\omega$*, in Open Problems in Topology. Ed. Jan van Mill and George M. Reed. Amsterdam: North-Holland, 1990. 97–125.

[3] Sergei Logunov, *On hereditary normality of zero-dimensional spaces*, Topology Appl. **102** (2000), no. 1, 53–58.

[4] B. È. Šapirovskiĭ, The imbedding of extremally disconnected spaces in bicompacta. $b$-points and weight of pointwise normal spaces. (Russian) Dokl. Akad. Nauk SSSR **223** (1975), no. 5, 1083–1086. English translation: Soviet Math. Dokl. **16** (1975), no. 4, 1056–1061 (1976).

[5] Jun Terasawa, *On the non-normality of $\beta X \setminus \{p\}$ for non-discrete spaces $X$*, Topology Proc. **27** (2003), no. 1, 335–344.

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