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THE CARTESIAN PRODUCT OF THE PSEUDO-ARC AND PSEUDO-CIRCLE IS FACTORWISE RIGID

KEVIN GAMMON

Abstract. The Cartesian product of two spaces is called factorwise rigid if every self homeomorphism can be written as a product homeomorphism. In 1983, D. Bellamy and J. Łysko proved that the Cartesian product of two pseudo-arcs is factorwise rigid. This argument relies on the fact that the pseudo-arc is chainable and does not easily generalize to products involving pseudo-circles. In this paper, the author proves that the Cartesian product of the pseudo-arc and pseudo-circle is factorwise rigid.

1. Introduction

A *continuum* is a nondegenerate, compact, and connected metric space. A continuum is *indecomposable* if it is not the union of two proper subcontinuum. A continuum is *hereditarily indecomposable* if every subcontinuum is indecomposable. Let X be a continuum and let x and y be elements of X. Then X is *chainable between x and y* provided that given $\epsilon > 0$, there is an ϵ chain covering X so that x is in the first link and y is in the last link. For more background on chains see, for example, [1] and [2].

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This paper will focus on the Cartesian product of two hereditarily indecomposable continua. The first, the pseudo-arc, was originally discovered by B. Knaster in 1922 [11]. A *pseudo-arc* is any hereditarily indecomposable chainable continuum. In 1948, E.E. Moise constructed a pseudo-arc as an indecomposable continuum homeomorphic to each of its nondegenerate subcontinua [16]. He was the first person to use the term pseudo-arc because the arc also has this property. Moise believed, but did not prove, that the hereditarily indecomposable continuum given by B. Knaster in 1922 was a pseudo-arc. In 1948, R.H. Bing proved that Moise's example was homogeneous [1]. In 1951, Bing proved that every hereditarily indecomposable chainable continuum is a pseudo-arc and that all pseudo-arcs are homeomorphic [2]. In an attempt to classify homogeneous planar continua, Bing gave another characterization of the pseudo-arc in 1959 as a homogeneous chainable continuum [3]. The pseudo-arc has been the subject of many interesting research questions. The history of many other aspects of the pseudo-arc can be found in survey papers by W. Lewis ([14] and [15]). Throughout this paper, P will denote a pseudo-arc which is chainable between two points α and β.

The second space examined is the pseudo-circle. In 1951, Bing [2] described the *pseudo-circle* as a planar, hereditarily indecomposable circularly chainable continuum which separates the plane. From this definition, it follows that every proper subcontinuum of the pseudo-circle is a pseudo-arc. Through a series of papers, L. Fearnley proved that the pseudo-circle is topologically unique ([6], [7], [9], [10]). Throughout this paper, C will denote the pseudo-circle.

A homeomorphism $h : X \times Y \to X \times Y$ is called a *product homeomorphism* if $h(x, y)$ can be written as

(1) $h(x, y) = (f(x), g(y))$ where $f : X \to X$ and $g : Y \to Y$ are homeomorphisms or
(2) $h(x, y) = (f(y), g(x))$ where $f : Y \to X$ and $g : X \to Y$ are homeomorphisms.

The Cartesian product $X \times Y$ of two continua is called *factorwise rigid* provided that if $h : X \times Y \to X \times Y$ is a homeomorphism, then h is a product homeomorphism.

It is known that the Cartesian product of two pseudo-arcs is factorwise rigid. This result is due to D. Bellamy and J. Łysko in [5].

Since the pseudo-arc and pseudo-circle share many properties, it was suspected that the result could be generalized to include pseudo-circles. However, the proof developed by D. Bellamy and J. Łysko relied on the fact that the pseudo-arc is chainable while the pseudo-circle does not have this property. In this paper we show that the Cartesian product of the pseudo-arc and the pseudo-circle is factorwise rigid.

2. Preliminary Information

In the following, the projection from a Cartesian product $A \times B$ to the first factor space will be denoted by π_1. Likewise, π_2 will denote the projection to the second factor space. $\check{H}_1(Y)$ will denote the first Čech homology group of the space Y.

Let G be a relation on $P \times P$ which collapses the fiber $P \times \{\alpha\}$ to a single point and $P \times \{\beta\}$ to a single point and consider the quotient space $(P \times P)/G$ with quotient map q. It is useful to notice that if $W \subset (P \times P)/G$ such that $q^{-1}(W)$ intersects $P \times \{\alpha\}$ (or $P \times \{\beta\}$), then $q^{-1}(W)$ contains $P \times \{\alpha\}$ (or $P \times \{\beta\}$.)

Lemma 2.1. *If $B \subset (P \times P)/G$ is a continuum, then $q^{-1}(B)$ is a continuum.*

Proof. $q^{-1}(B)$ is closed and hence compact because q is continuous. The fact that $q^{-1}(B)$ is connected is proven in Theorem 9, page 131 of [13]. □

Let $X = P \times C$. Then X can be essentially embedded inside of the cartesian product, Y, of the disk D^2 and an annulus A. Let $\widetilde{Y}$ denote the universal covering space of Y, which contains an infinite, connected covering space $\widetilde{X}$ of X. $\widehat{Y}$ will denote the two point compactification of $\widetilde{Y}$ obtained by adding points $\bar{a}$ and $\bar{b}$. Likewise, $\widehat{X}$ will denote the two points compactification of $\widetilde{X}$ contained inside of $\widehat{Y}$.

Lemma 2.2. *$(P \times P)/G$ is homeomorphic to $\widehat{X}$.*

Proof. In [4], D. Bellamy and W. Lewis have shown that the two point compactification of the infinite covering space, $\tilde{C}$, of the pseudo-circle obtained by unwrapping the pseudo-circle is a pseudo-arc. This implies that there is a homeomorphism f_1 from the covering space $\tilde{C}$ onto $P - \{\alpha, \beta\}$. Then the map $h_1(x, y) = (id_P(x), f_1(y))$, where id_P is the identity map on P, is a homeomorphism from $\widetilde{X}$ to $(P \times P) - (P \times \{\alpha\} \cup P \times \{\beta\})$. Then this map extends uniquely to a homeomorphism $H : \widehat{X} \to (P \times P)/G$. □

Let $g : X \to X$ be a homeomorphism. Then there exists a lift $\tilde{g}$ such that the following diagram commutes:

$$\begin{array}{ccc} & \widetilde{g} & \\ \widetilde{X} & \rightarrow & \widetilde{X} \\ \downarrow p & & \downarrow p \\ X & \rightarrow & X \\ & g & \end{array}$$

The argument that such a lift exists is similar to the lifting argument used by K. Kuperberg and the author in [12]. First note that since Y is an absolute neighborhood retract, g extends to a continuous map f from a closed, connected neighborhood of X homeomorphic to $D^2 \times A$ into $D^2 \times A$. Then $D^2 \times A$ can be retracted to this neighborhood of X. The composition of these maps has a lift, the appropriate restriction of this lift provides the lift of g.

Then $\widetilde{g}$ extends uniquely to a map $H : \widehat{X} \to \widehat{X}$. This map is a continuous bijection and hence a homeomorphism. Any such homeomorphism has the property that the set $\{a, b\}$ is invariant. In particular, since $\widehat{X}$ is homeomorphic to $(P \times P)/G$, the homeomorphism $g : X \to X$ uniquely induces a self homeomorphism of $(P \times P)/G$.

In this section, it will be shown that if $h : (P \times P)/G \to (P \times P)/G$ is such an induced homeomorphism then h has the additional properties that for any point $a \in P$

(1) $[q^{-1} \circ h \circ q](P \times \{a\}) = P \times \{b\}$ for some $b \in P$ and
(2) $[q^{-1} \circ h \circ q](\{a\} \times P) = (P \times \{\alpha\}) \cup (P \times \{\beta\}) \cup (\{b\} \times P)$ for some $b \in P$.

Throughout this section Φ will denote the set $(P \times \{\alpha\}) \cup (P \times \{\beta\})$. Notice that $q(\Phi)$ is an invariant set under the induced homeomorphism h.

The following Lemma in [5] will be needed:

Lemma 2.3. [Bellamy and Łysko, [5], Lemma 6] *Suppose M and N are indecomposable continua, and $a \in M$ and $h : M \times N \to M \times N$ is a homeomorphism. Then either $\pi_1(h(\{a\} \times N)) = M$ or $\pi_2(h(\{a\} \times N) = N$.*

The following Theorem of J. T. Rogers, Jr. will also be used:

Theorem 2.4. [Rogers, [17], Theorem 14] *The pseudo-circle is not the continuous image of the pseudo-arc.*

Lemma 2.5. *Let $a \in C$. Then $\pi_1(g(P \times \{a\})) = P$.*

Proof. Recall that $g : X \to X$ is a homeomorphism and notice that $\pi_2 \circ g(P \times \{a\}))$ is a continuous mapping of a pseudo-arc into a pseudo-circle. From Theorem 2.4, the pseudo-circle cannot be the continuous image of a pseudo-arc. Therefore that $\pi_2 \circ g(P \times \{a\}))$ cannot be onto. Thus, from Lemma 2.3, it follows that $\pi_1(g(P \times \{a\})) = P$. □

Lemma 2.6. *Let $a \in P$. Then $\pi_2(g(\{a\} \times C)) = C$.*

Proof. Since $\check{H}_1(P)$ is trivial, the restriction $g|_{\{a\}\times C} : \{a\} \times C \to P \times C$ induces an isomorphism between the groups $\check{H}_1(\{a\} \times C)$ and $\check{H}_1(P \times C)$. Likewise, since $\check{H}_1(P)$ is trivial, $\pi_2 : P \times C \to C$ induces an isomorphism between $\check{H}_1(P \times C)$ and $\check{H}_1(C)$. Therefore, the composition of these two maps induces an isomorphism between $\check{H}_1(\{a\} \times C)$ and $\check{H}_1(C)$. In particular, this implies that $\pi_2 \circ g(\{a\} \times C)$ must be onto. □

Since the homeomorphism $h : (P \times P)/G \to (P \times P)/G$ is uniquely determined by the homeomorphism $g : P \times C \to P \times C$, the following two corollaries are immediate from the previous two lemmas:

Corollary 2.7. $[\pi_1 \circ q^{-1} \circ h \circ q](P \times \{a\}) = P$ *for every* $a \in P$.

Corollary 2.8. *For every point $a \in P$, $[\pi_i \circ q^{-1} \circ h \circ q](\{a\} \times P) = P$ for $i \in \{1, 2\}$.*

For the following proofs it will be necessary to adapt a Lemma of Bellamy and Łysko in [5]:

Lemma 2.9. [Bellamy and Łysko, [5], Corollary 3] *Let M and N be chainable continua and suppose W and V are subcontinua of $M \times N$ such that $\pi_1(W) \subset \pi_1(V)$ while $\pi_2(V) \subset \pi_2(W)$. Then $W \cap V \neq \emptyset$.*

Lemma 2.10. *Suppose that W and M are subcontinua of $(P \times P)/G$ such that $\pi_1 \circ q^{-1}(W) \subset \pi_1 \circ q^{-1}(M)$ and $\pi_2 \circ q^{-1}(M) \subset \pi_2 \circ q^{-1}(W)$, then $M \cap N \neq \emptyset$.*

Proof. Since the inverses image under q of a continuum is a continuum, the inverse image satisfies the conditions of Lemma 2.9. □

With the previous Lemmas in mind, it will now be proven that the induced homeomorphism $h : (P \times P)/G \to (P \times P)/G$ has the additional properties that for any points $p \in P$

(1) $[q^{-1} \circ h \circ q](P \times \{p\}) = P \times \{a\}$ for some $a \in P$ and
(2) $[q^{-1} \circ h \circ q](\{p\} \times P) = \Phi \cup (\{b\} \times P)$ for some $b \in P$.

Theorem 2.11. *For every $p \in P$, $[q^{-1} \circ h \circ q](P \times \{p\}) = P \times \{b\}$ for some $b \in P$.*

Proof. If $p \in \{\alpha, \beta\}$, the result follows because the set $q(\Phi)$ is invariant under the homeomorphism h.

If $p \notin \{\alpha, \beta\}$, then the observations of the previous Lemmas allow the use of the proof of the main Theorem in [5] developed by Bellamy and Łysko. Suppose that $\pi_2(q^{-1} \circ h \circ q(P \times \{p\}))$ is non-degenerate. Let $\mathbb{Z}$ denote the set of non-negative integers and let $< W_n >_{n \in \mathbb{Z}}$ be a sequence of non-degenerate, decreasing subcontinua of P such that $\cap W_n = \{p\}$. Since this is a decreasing sequence, assume without loss of generality that $W_n \cap \{\alpha, \beta\} = \emptyset$ for each n. Let $a \in P$ and notice that

$$\bigcap(\{a\} \times W_n) = \{(a, p)\} \subset P \times \{p\},$$

therefore

$$\bigcap[\pi_2 \circ q^{-1} \circ h \circ q](\{a\} \times W_n) =$$

$$[\pi_2 \circ q^{-1} \circ h \circ q](a, p) \in [\pi_2 \circ q^{-1} \circ h \circ q](P \times \{p\}).$$

In particular, $[\pi_2 \circ q^{-1} \circ h \circ q](a, p)$ is an element of

$$[\pi_2 \circ q^{-1} \circ h \circ q](P \times \{p\}) \cap [\pi_2 \circ q^{-1} \circ h \circ q](\{a\} \times W_n)$$

for each n. Since P is hereditarily indecomposable, this implies that for each n either

(1) $[\pi_2 \circ q^{-1} \circ h \circ q](\{a\} \times W_n) \subset [\pi_2 \circ q^{-1} \circ h \circ q](P \times \{p\})$ or
(2) $[\pi_2 \circ q^{-1} \circ h \circ q](P \times \{p\}) \subset [\pi_2 \circ q^{-1} \circ h \circ q](\{a\} \times W_n)$.

Since $\cap([\pi_2 \circ q^{-1} \circ h \circ q](\{a\} \times W_n))$ is degenerate, condition (1) can not be true for each n. Therefore, there exists some N such that $[\pi_2 \circ q^{-1} \circ h \circ q](\{a\} \times W_N) \subset [\pi_2 \circ q^{-1} \circ h \circ q](P \times \{p\})$.

Let $x_1 \in W_N$ such that $x_1 \neq p$. From the above remarks, $[\pi_2 \circ q^{-1} \circ h \circ q](P \times \{x_1\}) \cap [\pi_2 \circ q^{-1} \circ h \circ q](P \times \{p\}) \neq \emptyset$.

This implies that either

(1) $[\pi_2 \circ q^{-1} \circ h \circ q](P \times \{x_1\}) \subset [\pi_2 \circ q^{-1} \circ h \circ q](P \times \{p\})$ or
(2) $[\pi_2 \circ q^{-1} \circ h \circ q](P \times \{p\}) \subset [\pi_2 \circ q^{-1} \circ h \circ q](P \times \{x_1\})$.

We will prove the first case, the proof of the second case is similar. Notice from Lemma 2.7, $[\pi_1 \circ q^{-1} \circ h \circ q](P \times \{x_1\}) = P = [\pi_1 \circ q^{-1} \circ h \circ q](P \times \{p\})$, therefore the conditions of Lemma 2.10 are satisfied. Hence $[h \circ q](P \times \{x_1\}) \cap [h \circ q](P \times \{p\}) \neq \emptyset$. However, this is a contradiction since $[q^{-1} \circ h \circ q]$ restricted to $(P \times P) - \Phi$ is a homeomorphism. □

Theorem 2.12. *$[q^{-1} \circ h \circ q](\{a\} \times P) = \Phi \cup (\{b\} \times P)$ for some $b \in P$.*

Proof. Let $x \in P$ such that $K(x)$ does not contain the set $\{\alpha, \beta\}$. Such a point exists because an indecomposable continuum has uncountably many pairwise disjoint composants (see, for example, K. Kuratowski, [13], Theorems 5 and 7, p. 212). It will first be shown that $[q^{-1} \circ h \circ q](\{a\} \times K(x)) \subset \{b\} \times P$ for some $b \in P$.

Let P_1 be a nondegenerate subcontinuum of $K(x)$. Note that P_1 is a pseudo-arc and consider the subcontinuum of $P \times P_1$ of $P \times P$. From Lemma 2.11, for every point $x_1 \in P_1$, the map $[q^{-1} \circ h \circ q](P \times \{x_1\})$ is mapped homeomorphically onto $P \times \{x_2\}$ for some $x_2 \in P$. Note that x_2 can not equal α or β. In particular, $[q^{-1} \circ h \circ q](P \times P_1)$ is mapped bijectively onto $P \times P_2$ where P_2 is a proper, nondegenerate subcontinuum of P and therefore a pseudo-arc. Similar to the proof of Theorem 2.11, the proof of the main result by Bellamy and Łysko in [5] can be applied to show that $[q^{-1} \circ h \circ q]$ restricted to $P \times P_1$ also preserves horizontal fibers. In particular, $[q^{-1} \circ h \circ q](\{a\} \times P_1) \subset \{b\} \times P$ for some $b \in P$.

Let $p \in P$ and consider $\{p\} \times P_1$. Let $< W_n >_{n \in \mathbb{Z}}$ be sequence of decreasing, non-degenerate subcontinua of P such that $\cap W_n = \{p\}$. Next, Let $a \in P_1$ and notice that

$$\cap (W_n \times \{a\}) = \{(p, a)\} \in \{p\} \times P_1.$$

In particular, $\cap [\pi_1 \circ q^{-1} \circ h \circ q](W_n \times \{a\}) = [\pi_1 \circ q^{-1} \circ h \circ q](p, a)$ is an element of $[\pi_1 \circ q^{-1} \circ h \circ q](\{p\} \times P_1)$.

Therefore,

$$[\pi_1 \circ q^{-1} \circ h \circ q](W_n \times \{a\}) \cap [\pi_1 \circ q^{-1} \circ h \circ q](\{p\} \times P_1) \neq \emptyset$$

for each n. This implies that for each n, either

(1) $[\pi_1 \circ q^{-1} \circ h \circ q](W_n \times \{a\}) \subset [\pi_1 \circ q^{-1} \circ h \circ q](\{p\} \times P_1)$ or
(2) $[\pi_1 \circ q^{-1} \circ h \circ q](\{p\} \times P_1) \subset [\pi_1 \circ q^{-1} \circ h \circ q](W_n \times \{a\})$.

However, since $\cap([\pi_1 \circ q^{-1} \circ h \circ q](W_n \times \{a\}))$ is degenerate, condition 2 cannot hold for every n. Thus, there exists some N so that $[\pi_1 \circ q^{-1} \circ h \circ q](W_N \times \{a\}) \subset [\pi_1 \circ q^{-1} \circ h \circ q](\{p\} \times P_1)$.

Let $x_1 \in W_N$ such that $x_1 \neq p$. From the above remarks, $[\pi_1 \circ q^{-1} \circ h \circ q](\{x_1\} \times P_1) \cap [\pi_1 \circ q^{-1} \circ h \circ q](\{p\} \times P_1) \neq \emptyset$. Since the pseudo-arc is hereditarily indecomposable this implies that either

(1) $[\pi_1 \circ q^{-1} \circ h \circ q](\{x_1\} \times P_1) \subset [\pi_1 \circ q^{-1} \circ h \circ q](\{p\} \times P_1)$ or
(2) $[\pi_1 \circ q^{-1} \circ h \circ q](\{p\} \times P_1) \subset [\pi_1 \circ q^{-1} \circ h \circ q](\{x_1\} \times P_1)$.

The first case will be proven, the second case is similar. Since $[\pi_2 \circ q^{-1} \circ h \circ q](\{x_1\} \times P_1) = P_2 = [\pi_2 \circ q^{-1} \circ h \circ q](\{p\} \times P_1)$, the conditions of Lemma 2.10 are satisfied. Therefore, $[h \circ q](\{x_1\} \times P_1) \cap [h \circ q](\{p\} \times P_1) \neq \emptyset$. This contradicts the fact that $[q^{-1} \circ h \circ q]$ restricted to $(P \times P) - \Phi$ is a homeomorphism. Therefore $[q^{-1} \circ h \circ q](\{a\} \times P_1) \subset \{b\} \times P$ for some $b \in P$.

Next, notice that since P is hereditarily indecomposable any two points in $K(x)$ can be joined by a proper subcontinuum. Therefore, $[q^{-1} \circ h \circ q](\{a\} \times K(x)) \subset \{b\} \times P$.

However, note that $h \circ q(\{a\} \times P) = h \circ q(cl(\{a\} \times K(x))$, since composants in an indecomposable space are dense. From the previous paragraphs, this implies that $h \circ q(\{a\} \times P) = q(\{b\} \times P)$. Therefore it follows that $[q^{-1} \circ h \circ q](\{a\} \times P) = (\{b\} \times P) \cup \Phi$. □

3. Factorwise rigidity of $P \times C$

As in the previous section, let $X = P \times C$ and let $\widehat{X}$ will denote the two points compactification of the infinite covering space $\widetilde{X}$ of X.

Theorem 3.1. *The Cartesian product $P \times C$ is factorwise rigid.*

Proof. Let $h : X \to X$ be a homeomorphism. Then there exists a lift $\tilde{h}$ such that the following diagram commutes:

$$\begin{array}{ccc} & \tilde{h} & \\ \widetilde{X} & \rightarrow & \widetilde{X} \\ \downarrow p & & \downarrow p \\ X & \rightarrow & X \\ & h & . \end{array}$$

Then $\widetilde{h}$ extends uniquely to a map $H : \widehat{X} \to \widehat{X}$. This map is a continuous bijection and hence a homeomorphism. Any such homeomorphism has the property that the set $\{a, b\}$ is invariant. Note that $\widehat{X}$ is homeomorphic to $(P \times P)/G$ and therefore the results of the previous section apply. In particular, for $x \in C$, $h(P \times \{x\}) = p \circ H \circ p^{-1}(P \times \{x\})$. However, from Theorem 2.11, $H \circ p^{-1}(P \times \{x\}) = p^{-1}(P \times \{y\})$ for some $y \in C$. Hence $h(P \times \{x\}) = P \times \{y\}$.

Likewise, from Theorem 2.12, it follows that $h(\{r\} \times C) = p \circ H \circ p^{-1}(\{r\} \times C) = \{s\} \times C$ for some $s \in P$.

Therefore, the cartesian product $P \times C$ is factorwise rigid. □

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