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ON Σ-PONOMAREV-SYSTEMS

NGUYEN VAN DUNG

Abstract. In this paper, we introduce the Σ-Ponomarev-system $(f, M, X, \{\mathcal{P}_n^*\})$ to investigate relations between the mapping f from a metric space M onto a space X with the σ-network $\bigcup\{\mathcal{P}_n^* : n \in \mathbb{N}\}$. By this system, we give necessary and sufficient conditions for f to be an (a) s-(compact, $msss$-, $mssc$-, cs-)mapping, and a sufficient condition for f to be a π-mapping. Also, we give necessary and sufficient conditions for f to be a mapping with covering-properties.

1. Introduction

Finding characterizations of nice images of metric spaces is one of the most important problems in general topology. Related to this problem, various kinds of characterizations of s-(resp., compact, π-, $msss$-, $mssc$-, cs-)images of metric spaces have been obtained by means of certain point-countable (resp., point-finite σ-strong, σ-strong, σ-locally countable, σ-locally finite, compact-countable) networks [14], [20]. The key to prove these results is to construct s-(resp., compact, π-, $msss$-, $mssc$-, cs-)mappings with covering-properties from metric spaces onto spaces with certain networks. In [16], S. Lin and P. Yan introduced Ponomarev-systems $(f, M, X, \mathcal{P})$ and $(f, M, X, \{\mathcal{P}_n\})$ to give general conditions for f to be a compact-covering mapping onto a space X from some metric space M, where $\mathcal{P}$ is a strong network and $\bigcup\{\mathcal{P}_n : n \in \mathbb{N}\}$ is a

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σ-strong network for X. After that, Y. Ge and S. Lin have been obtained necessary and sufficient conditions for f to be an s-mapping with covering-properties in a Ponomarev-system $(f, M, X, \mathcal{P})$ [6], [8], [9]. Also, the authors have been obtained necessary and sufficient conditions for f to be a compact mapping with covering-properties in a Ponomarev-system $(f, M, X, \{\mathcal{P}_n\})$ [7], [8]. Note that, in a Ponomarev-system $(f, M, X, \{\mathcal{P}_n\})$, the mapping f is a π-mapping [16]. So far it seems not to be known whether a necessary and sufficient condition for f to be a compact mapping can be obtained in a Ponomarev-system $(f, M, X, \mathcal{P})$. Also, we do not know whether necessary and sufficient conditions for f to be an (a) $msss$-($mssc$-, cs-)mapping with covering-properties can be obtained in Ponomarev-systems $(f, M, X, \mathcal{P})$ and $(f, M, X, \{\mathcal{P}_n\})$.

By the above, we are interested in finding a general system to give necessary and sufficient conditions for the mapping f to be an (a) s-(compact, π-, $msss$-, $mssc$-, cs-)mapping with covering-properties from some metric space M onto a space X with certain networks.

In this paper, we introduce the notion of a Σ-Ponomarev-system $(f, M, X, \{\mathcal{P}_n^*\})$ to investigate relations between the mapping f from a metric space M onto a space X with the σ-network $\bigcup\{\mathcal{P}_n^* : n \in \mathbb{N}\}$. By this system, we give necessary and sufficient conditions for f to be an (a) s-(compact, $msss$-, $mssc$-, cs-) mapping, and a sufficient condition for f to be a π-mapping. Also, we give necessary and sufficient conditions for f to be a mapping with covering-properties.

Throughout this paper, all spaces are Hausdorff, all mappings are continuous and onto, a convergent sequence includes its limit point, $\mathbb{N}$ denotes the set of all natural numbers, $\omega = \mathbb{N} \cup \{0\}$, and p_k denotes the projection of $\prod_{n\in\mathbb{N}} X_n$ onto X_k. Let X be a space, $x \in X$, and $\mathcal{P}$ be a family of subsets of X, we denote $st(x, \mathcal{P}) = \bigcup\{P \in \mathcal{P} : x \in P\}$. We say that a convergent sequence $\{x_n : n \in \mathbb{N}\} \cup \{x\}$ converging to x is *eventually* in A if $\{x_n : n \geq n_0\} \cup \{x\} \subset A$ for some $n_0 \in \mathbb{N}$.

For terms are not defined here, please refer to [3] and [14].

2. Main results

Definition 2.1. Let $f : X \longrightarrow Y$ be a mapping.

(1) f is an *msss-mapping* [12], if X is a subspace of the product space $\prod_{n\in\mathbb{N}} X_n$ of a family $\{X_n : n \in \mathbb{N}\}$ of metric spaces, and for each $y \in Y$, there exists a sequence $\{V_{y,n} : n \in \mathbb{N}\}$ of open neighborhoods of y in Y such that each $p_n(f^{-1}(V_{y,n}))$ is a separable subset of X_n.

(2) f is an *mssc-mapping* [12], if X is a subspace of the product space $\prod_{n\in\mathbb{N}} X_n$ of a family $\{X_n : n \in \mathbb{N}\}$ of metric spaces, and for each $y \in Y$, there exists a sequence $\{V_{y,n} : n \in \mathbb{N}\}$ of open neighborhoods of y in Y such that each $\overline{p_n(f^{-1}(V_{y,n}))}$ is a compact subset of X_n.

(3) f is an *s-mapping* (resp., *compact* mapping) [1], if for each $y \in Y$, $f^{-1}(y)$ is a separable (resp., compact) subset of X.

(4) f is a *cs-mapping* [18] if, for each compact subset K of Y, $f^{-1}(K)$ is a separable subset of X.

(5) f is a *π-mapping* [1], if for each $y \in Y$ and each neighborhood U of y in Y, $d(f^{-1}(y), X - f^{-1}(U)) > 0$, where X is a metric space with a metric d.

(6) f is an *1-sequence-covering* mapping [13] if, for each $y \in Y$, there exists $x_y \in f^{-1}(y)$ such that whenever $\{y_n : n \in \mathbb{N}\}$ is a sequence converging to y in Y there exists a sequence $\{x_n : n \in \mathbb{N}\}$ converging to x_y in X with each $x_n \in f^{-1}(y_n)$.

(7) f is a *2-sequence-covering* mapping [13] if, for each $y \in Y$, $x_y \in f^{-1}(y)$, and sequence $\{y_n : n \in \mathbb{N}\}$ converging to y in Y, there exists a sequence $\{x_n : n \in \mathbb{N}\}$ converging to x_y in X with each $x_n \in f^{-1}(y_n)$.

(8) f is a *sequence-covering* mapping [19] if, for each convergent sequence S in Y, there exists a convergent sequence L in X such that $f(L) = S$. Note that a sequence-covering mapping is a *strong sequence-covering* mapping in the sense of [11].

(9) f is a *subsequence-covering* mapping [15] if, for each convergent sequence S in Y, there exists a compact subset K of X such that $f(K)$ is a subsequence of S.

(10) f is a *sequentially-quotient* mapping [2] if, for each convergent sequence S in Y, there exists a convergent sequence L in X such that $f(L)$ is a subsequence of S.

Definition 2.2. Let P be a subset of a space X. P is a *sequential neighborhood* of x [4], if for every convergent sequence S converging to x in X, S is eventually in P.

Definition 2.3. Let $\mathcal{P}$ be a family of subsets of a space X.

(1) $\mathcal{P}$ is a *network at* x in X [17], if $x \in P$ for every $P \in \mathcal{P}$, and whenever $x \in U$ with U open in X, then there exists $P \in \mathcal{P}$ such that $x \in P \subset U$.

(2) $\mathcal{P}$ is a *strong network* for X if, for every $x \in X$, there exists $\mathcal{P}_x \subset \mathcal{P}$ such that $\mathcal{P}_x$ is a countable network at x in X.

Definition 2.4. Let $\mathcal{P} = \bigcup\{\mathcal{P}_n : n \in \mathbb{N}\}$, where each $\mathcal{P}_n$ is a family of subsets a space X.

(1) $\mathcal{P}_x = \{P_{\alpha_n} : n \in \mathbb{N}\} \subset \mathcal{P}$ is a *σ-network at* x in X, if $\mathcal{P}_x$ is a network at x in X, and $P_{\alpha_n} \in \mathcal{P}_n$ for every $n \in \mathbb{N}$.

$\mathcal{P}$ is a *σ-network* for X, if for each $x \in X$, there exists $\mathcal{P}_x \subset \mathcal{P}$ such that $\mathcal{P}_x$ is a σ-network at x in X.

(2) $\mathcal{P}$ is a *σ-strong network* for X [10], if $\{st(x, \mathcal{P}_n) : n \in \mathbb{N}\}$ is a network at x in X for every $x \in X$.

(3) $\mathcal{P}$ is a *σ-cs-network* for X, if whenever S is a convergent sequence converging to x in X there exists $\{P_{\alpha_n} : n \in \mathbb{N}\} \subset \mathcal{P}$ such that $\{P_{\alpha_n} : n \in \mathbb{N}\}$ is a σ-network at x in X, and S is eventually in P_{α_n} for every $n \in \mathbb{N}$.

(4) $\mathcal{P}$ is a *σ-cs*-network* for X, if whenever S is a convergent sequence in X there exists a subsequence L of S and $\{P_{\alpha_n} : n \in \mathbb{N}\} \subset \mathcal{P}$ such that $\{P_{\alpha_n} : n \in \mathbb{N}\}$ is a σ-network at x in X, and L is eventually in P_{α_n} for every $n \in \mathbb{N}$.

(5) $\mathcal{P}$ is a *σ-sn-network* for X, if for each $x \in X$ there exists $\{P_{\alpha_n} : n \in \mathbb{N}\} \subset \mathcal{P}$ such that $\{P_{\alpha_n} : n \in \mathbb{N}\}$ is a σ-network at x in X, and P_{α_n} is a sequential neighborhood of x for every $n \in \mathbb{N}$.

(6) $\mathcal{P}$ is a *σ-so-network* for X, if for each $x \in X$ there exists $\{P_{\alpha_n} : n \in \mathbb{N}\} \subset \mathcal{P}$ such that $\{P_{\alpha_n} : n \in \mathbb{N}\}$ is a σ-network at x in X, and whenever $\{P_{\alpha_n} : n \in \mathbb{N}\} \subset \mathcal{P}$ is a σ-network at x in X, then P_{α_n} is a sequential neighborhood of x for every $n \in \mathbb{N}$.

(7) $\mathcal{P}$ is a *σ-strong cs-*(resp., *σ-strong cs*-*, *σ-strong sn-*, *σ-strong so-*)*network* for X, if $\mathcal{P}$ is a σ-strong network and a σ-*cs*-(resp., σ-*cs**-, σ-*sn*-, σ-*so*-)network for X.

(8) $\mathcal{P}$ is a *σ-(P)* (resp., *σ-(P) strong*, *σ-(P) (p)-*, *σ-(P) strong (p)-*)*network* for X, if $\mathcal{P}$ is a σ-(resp., σ-strong, σ-(p)-, σ-strong (p)-)network and each $\mathcal{P}_n$ has property (P), where (P) is point-countable (point-finite, locally countable, locally finite, compact-countable), and (p) is *cs*- (*cs**-, *sn*-, *so*-).

Definition 2.5. (1) Let $\mathcal{P}$ be a strong network for a space X. Put $\mathcal{P} = \{P_\alpha : \alpha \in A\}$. For every $n \in \mathbb{N}$, put $A_n = A$ and endow A_n with a discrete topology. Put

$$M = \big\{a = (\alpha_n) \in \prod_{n\in\mathbb{N}} A_n : \{P_{\alpha_n} : n \in \mathbb{N}\} \text{ forms a network at some point } x_a \text{ in } X\big\}.$$

Then M, which is a subspace of the product space $\prod_{n\in\mathbb{N}} A_n$, is a metric space, x_a is unique, and $x_a = \bigcap_{n\in\mathbb{N}} P_{\alpha_n}$ for every $a \in M$. Define $f : M \to X$ by $f(a) = x_a$, then f is a mapping, and $(f, M, X, \mathcal{P})$ is a *Ponomarev-system* [16].

(2) Let $\bigcup\{\mathcal{P}_n : n \in \mathbb{N}\}$ be a σ-network for X. For every $n \in \mathbb{N}$, put $\mathcal{P}_n = \{P_\alpha : \alpha \in A_n\}$ and endow A_n with a discrete topology. Put

$$M = \big\{a = (\alpha_n) \in \prod_{n\in\mathbb{N}} A_n : \{P_{\alpha_n} : n \in \mathbb{N}\} \text{ forms a network at some point } x_a \text{ in } X\big\}.$$

Then M, which is a subspace of the product space $\prod_{n\in\mathbb{N}} A_n$, is a metric space, x_a is unique, and $x_a = \bigcap_{n\in\mathbb{N}} P_{\alpha_n}$ for every $a \in M$. Define $f : M \to X$ by $f(a) = x_a$, then f is a mapping, and $(f, M, X, \{\mathcal{P}_n\})$ is a *Ponomarev-system* [21]. Under $\bigcup\{\mathcal{P}_n : n \in \mathbb{N}\}$ being a σ-strong network for X, $(f, M, X, \{\mathcal{P}_n\})$ is a *Ponomarev-system* in the sense of [16].

Definition 2.6. Let $\mathcal{P} = \bigcup\{\mathcal{P}_n : n \in \mathbb{N}\}$ be a σ-network for X. For every $n \in \mathbb{N}$, put $\mathcal{P}_n^* = \mathcal{P}_n \cup \{\bigcap_{i=1}^n P_i : P_i \in \mathcal{P}_i, i \le n\} = \{P_\alpha : \alpha \in A_n\}$ and endow A_n with a discrete topology. Put

$$M = \big\{a = (\alpha_n) \in \prod_{n\in\mathbb{N}} A_n : \{P_{\alpha_n} : n \in \mathbb{N}\} \text{ forms a network at some point } x_a \text{ in } X\big\}.$$

Then, M is a metric space, x_a is unique, and $x_a = \bigcap_{n\in\mathbb{N}} P_{\alpha_n}$ for every $a \in M$. Define $f : M \longrightarrow X$ by $f(a) = x_a$, then f is a mapping by the following Lemma 2.7, and $(f, M, X, \{\mathcal{P}_n^*\})$ is a Σ-*Ponomarev-system.*

Lemma 2.7. *Let $(f, M, X, \{\mathcal{P}_n^*\})$ be the system in Definition 2.6. Then the following hold.*

(1) *f is onto.*
(2) *f is continuous.*

Proof. (1). For each $x \in X$, since $\mathcal{P}$ is a σ-network for X, there exists $P_{\alpha_n} \in \mathcal{P}_n$ for every $n \in \mathbb{N}$ such that $\{P_{\alpha_n} : n \in \mathbb{N}\}$ forms a network at x in X. Put $a = (\alpha_n)$, then $a \in M$ and $f(a) = x$. This proves that f is onto.

(2). For each $a = (\alpha_n) \in M$ and $f(a) = x_a$, let V be an open neighborhood of x_a in X. Then there exists $k \in \mathbb{N}$ such that $x_a \in P_{\alpha_k} \subset V$. Put $U = \{b = (\beta_n) \in M : \beta_k = \alpha_k\}$. Then U is an open neighborhood of a in M and $f(U) \subset P_{\alpha_k} \subset V$. It implies that f is continuous. $\square$

Remark 2.8. (1) For each $n \in \mathbb{N}$, $\mathcal{P}_n$ has property (P) if and only if $\mathcal{P}_n^*$ has property (P), where (P) is point-countable (point-finite, locally countable, locally finite, compact-countable).

(2) $\bigcup\{\mathcal{P}_n : n \in \mathbb{N}\}$ is a σ-strong network for X if and only if $\bigcup\{\mathcal{P}_n^* : n \in \mathbb{N}\}$ is a σ-strong network for X.

(3) If $\bigcup\{\mathcal{P}_n : n \in \mathbb{N}\}$ is a σ-network (resp., σ-*cs*-network, σ-*cs**-network, σ-*sn*-network, σ-*so*-network) for X, then $\bigcup\{\mathcal{P}_n^* : n \in \mathbb{N}\}$ is a σ-network (resp., σ-*cs*-network, σ-*cs**-network, σ-*sn*-network, σ-*so*-network) for X.

(4) The Σ-Ponomarev-system $(f, M, X, \{\mathcal{P}_n^*\})$ where X is a space with σ-network $\bigcup\{\mathcal{P}_n : n \in \mathbb{N}\}$ is the Ponomarev-system $(f, M, X, \{\mathcal{P}_n^*\})$ where X is a space with σ-network $\bigcup\{\mathcal{P}_n^* : n \in \mathbb{N}\}$.

Lemma 2.9. *Let $(f, M, X, \{\mathcal{P}_n^*\})$ be a Σ-Ponomarev-system, $a = (\alpha_n) \in M$ where $\{P_{\alpha_n} : n \in \mathbb{N}\}$ is a network at some point x_a in X, and*

$$U_n = \{b = (\beta_i) \in M : \beta_i = \alpha_i \text{ if } i \leq n\},$$

for every $n \in \mathbb{N}$. Then the following hold.

(1) *$\{U_n : n \in \mathbb{N}\}$ is a base at a in M.*
(2) *$f(U_n) = \bigcap_{i=1}^n P_{\alpha_i}$ for every $n \in \mathbb{N}$.*

Proof. (1). It is obvious.

(2). For each $n \in \mathbb{N}$, let $x \in f(U_n)$. Then $x = f(b)$ for some $b = (\beta_i) \in U_n$. Therefore, $x = \bigcap_{i \in \mathbb{N}} P_{\beta_i} \subset \bigcap_{i=1}^n P_{\beta_i} = \bigcap_{i=1}^n P_{\alpha_i}$. This proves that $f(U_n) \subset \bigcap_{i=1}^n P_{\alpha_i}$.

Conversely, let $x \in \bigcap_{i=1}^n P_{\alpha_i}$. Then $x = f(b)$ for some $b = (\beta_i) \in M$. Put $c = (\gamma_i)$, where $P_{\gamma_i} = P_{\alpha_i}$ if $i \leq n$ and $P_{\gamma_i} = \bigcap_{j=1}^i P_{\beta_j}$ if $i > n$. Then $c \in U_n$ and $f(c) = x$. It implies that $\bigcap_{i=1}^n P_{\alpha_i} \subset f(U_n)$.

By the above, we get $f(U_n) = \bigcap_{i=1}^n P_{\alpha_i}$. □

In [6], [8], Y. Ge and S. Lin obtained necessary and sufficient conditions such that f is an s-mapping for the Ponomarev-system $(f, M, X, \mathcal{P})$ and f is a compact mapping for the Ponomarev-system $(f, M, X, \{\mathcal{P}_n\})$. In the following, we give necessary and sufficient conditions for f to be an (a) s-(compact, *mssc*-, *msss*-, *cs*-) mapping in a Σ-Ponomarev-system $(f, M, X, \{\mathcal{P}_n^*\})$.

Theorem 2.10. *Let $(f, M, X, \{\mathcal{P}_n^*\})$ be a Σ-Ponomarev-system. Then the following hold.*

(1) *f is an s-mapping if and only if $\bigcup\{\mathcal{P}_n^* : n \in \mathbb{N}\}$ is a σ-point-countable network for X.*
(2) *f is a compact mapping if and only if $\bigcup\{\mathcal{P}_n^* : n \in \mathbb{N}\}$ is a σ-point-finite strong network for X.*
(3) *f is an msss-mapping if and only if $\bigcup\{\mathcal{P}_n^* : n \in \mathbb{N}\}$ is a σ-locally countable network for X.*
(4) *f is an mssc-mapping if and only if $\bigcup\{\mathcal{P}_n^* : n \in \mathbb{N}\}$ is a σ-locally finite network for X.*
(5) *f is a cs-mapping if and only if $\bigcup\{\mathcal{P}_n^* : n \in \mathbb{N}\}$ is a σ-compact-countable network for X.*

Proof. (1). *Necessity.* Let f be an s-mapping. If there exists $k \in \mathbb{N}$ such that $\mathcal{P}_k^*$ is not point-countable, then, for some $x \in X$, we have that $A_{x,k} = \{\alpha \in A_k : x \in P_\alpha\}$ is uncountable. For each $\alpha \in A_{x,k}$, put $U_\alpha = \{b = (\beta_n) \in M : \beta_k = \alpha\}$, then U_α is open. If $b = (\beta_n) \in f^{-1}(x)$, then $x = f(b) \in P_{\beta_k}$. It implies that $\beta_k = \alpha$ for some $\alpha \in A_{x,k}$, hence $b \in U_\alpha$. Therefore, $\{U_\alpha : \alpha \in A_{x,k}\}$ is an uncountable open cover for $f^{-1}(x)$, but it has not any proper subcover. So $f^{-1}(x)$ is not separable, hence f is not an s-mapping. It is a contradiction.

Sufficiency. Let $\bigcup\{\mathcal{P}_n^* : n \in \mathbb{N}\}$ be a σ-point-countable network for X. Then for each $x \in X$, we have that $A_{x,n} = \{\alpha \in A_n : x \in P_\alpha\}$ is countable for every $n \in \mathbb{N}$. Therefore, $\prod_{n\in\mathbb{N}} A_{x,n}$ is hereditarily separable. It follows from $f^{-1}(x) \subset \prod_{n\in\mathbb{N}} A_{x,n}$ that $f^{-1}(x)$ is separable. Then f is an s-mapping.

(2). *Necessity.* Let f be a compact mapping. It suffices to prove following claims (a) and (b).

(a) $\bigcup\{\mathcal{P}_n^* : n \in \mathbb{N}\}$ is a σ-strong network for X.

If $\bigcup\{\mathcal{P}_n^* : n \in \mathbb{N}\}$ is not a σ-strong network for X. Then there exists $x \in U$ with U open in X such that $st(x, \mathcal{P}_n^*) \not\subset U$ for every $n \in \mathbb{N}$. Therefore, there exists $P_{\alpha_n} \in \mathcal{P}_n^*$ such that $x \in P_{\alpha_n} \not\subset U$ for every $n \in \mathbb{N}$. It implies that $\{P_{\alpha_n} : n \in \mathbb{N}\}$ does not form a network at x in X, hence $a = (\alpha_n) \notin f^{-1}(x)$. Let $x = f(b)$ for some $b = (\beta_n) \in M$. For each $i \in \mathbb{N}$, put $a_i = (\alpha_{in})$, where $P_{\alpha_{in}} = P_{\alpha_n}$ if $n \leq i$ and $P_{\alpha_{in}} = \bigcap_{j=1}^n P_{\beta_j}$ if $n > i$. Then $\{P_{\alpha_{in}} : n \in \mathbb{N}\}$ forms a network at x in X, so $a_i \in f^{-1}(x)$ for every $i \in \mathbb{N}$. It is easy to see that the sequence $\{a_i : i \in \mathbb{N}\}$ converges to a in $\prod_{n\in\mathbb{N}} A_n$. Since $f^{-1}(x)$ is a compact subset of M, $f^{-1}(x)$ is closed in $\prod_{n\in\mathbb{N}} A_n$. Then $a \in f^{-1}(x)$. It is a contradiction.

(b) Each $\mathcal{P}_n^*$ is point-finite.

If there exists $k \in \mathbb{N}$ such that $\mathcal{P}_k^*$ is not point-finite, then, for some $x \in X$, we have that $A_{x,k} = \{\alpha \in A_k : x \in P_\alpha\}$ is infinite. For each $\alpha \in A_{x,k}$, put $U_\alpha = \{b = (\beta_n) \in M : \beta_k = \alpha\}$, then U_α is open. If $b = (\beta_n) \in f^{-1}(x)$, then $x = f(b) \in P_{\beta_k}$. It implies that $\beta_k = \alpha$ for some $\alpha \in A_{x,k}$, hence $b \in U_\alpha$. Therefore, $\{U_\alpha : \alpha \in A_{x,k}\}$ is an infinite open cover for $f^{-1}(x)$, but it has not any proper subcover. So $f^{-1}(x)$ is not compact, hence f is not a compact mapping. It is a contradiction.

Sufficiency. Let $\bigcup\{\mathcal{P}_n^* : n \in \mathbb{N}\}$ be a σ-point-finite strong network for X. Then for each $x \in X$, we have that $A_{x,n} = \{\alpha \in A_n : x \in P_\alpha\}$ is finite and $\{P_{\alpha_n} : n \in \mathbb{N}\}$ forms a network at x in X for every $(\alpha_n) \in \prod_{n\in\mathbb{N}} A_{x,n}$. Therefore, $f^{-1}(x) = \prod_{n\in\mathbb{N}} A_{x,n}$, and so f is a compact mapping.

(3). *Necessity.* Let f be an $msss$-mapping. If there exists $k \in \mathbb{N}$ such that $\mathcal{P}_k^*$ is not locally countable, then, for some $x \in X$, we have that $A_{x,k} = \{\alpha \in A_k : P_\alpha \cap U_x \neq \emptyset\}$ is uncountable for every open neighborhood U_x of x in X. For each $\alpha \in A_{x,k}$, pick $y \in P_\alpha \cap U_x$, and let $y = f(a)$ for some $a = (\alpha_n) \in M$. Put $b_\alpha = (\beta_n)$, where $P_{\beta_k} = P_\alpha$, and $P_{\beta_n} = \bigcap_{j=1}^n P_{\alpha_j}$ if $n \neq k$. Then $\beta_n \in A_n$ for every $n \in \mathbb{N}$ and $\{P_{\beta_n} : n \in \mathbb{N}\}$ forms a network at y in X. So $b_\alpha \in f^{-1}(y) \subset f^{-1}(U_x)$. It implies that $\alpha = p_k(b_\alpha) \in p_k(f^{-1}(U_x))$.

Then $A_{x,k} \subset p_k(f^{-1}(U_x)) \subset A_k$. Since $A_{x,k}$ is uncountable and A_k is discrete, $p_k(f^{-1}(U_x))$ is not separable. It is a contradiction.

Sufficiency. Let $\bigcup\{\mathcal{P}_n^* : n \in \mathbb{N}\}$ be a σ-locally countable network for X. For each $x \in X$ and $n \in \mathbb{N}$, there exists an open neighborhood $U_{x,n}$ of x in X such that $A_{x,n} = \{\alpha \in A_n : P_\alpha \cap U_{x,n} \neq \emptyset\}$ is countable. It implies that $f^{-1}(U_{x,n}) \subset \prod_{n\in\mathbb{N}} A_{x,n}$, then $p_n(f^{-1}(U_{x,n})) \subset A_{x,n}$. Since $A_{x,n}$ is countable, $p_n(f^{-1}(U_{x,n}))$ is separable. Then f is an *msss*-mapping.

(4). *Necessity.* Let f be an *mssc*-mapping. If there exists $k \in \mathbb{N}$ such that $\mathcal{P}_k^*$ is not locally finite, then, by using notations and arguments in the necessity of (3) again, we have that $A_{x,k}$ is infinite and $A_{x,k} \subset p_k(f^{-1}(U_x))$. This proves that $\overline{p_k(f^{-1}(U_x))}$ is not compact. It is a contradiction.

Sufficiency. Let $\bigcup\{\mathcal{P}_n^* : n \in \mathbb{N}\}$ be a σ-locally finite network for X. By using notations and arguments in the sufficiency of (3) again, we have that $A_{x,n}$ is finite and $p_n(f^{-1}(U_{x,n})) \subset A_{x,n}$ for every $n \in \mathbb{N}$. Then $\overline{p_n(f^{-1}(U_{x,n}))}$ is compact. This proves that f is an *mssc*-mapping.

(5). *Necessity.* Let f be a *cs*-mapping. If there exists $k \in \mathbb{N}$ such that $\mathcal{P}_k^*$ is not compact-countable, then, for some compact subset C of X, we have that $A_{C,k} = \{\alpha \in A_k : C \cap P_\alpha \neq \emptyset\}$ is uncountable. For each $\alpha \in A_{C,k}$, put $U_\alpha = \{b = (\beta_n) \in M : \beta_k = \alpha\}$, then U_α is open. If $b = (\beta_n) \in f^{-1}(C)$, then $x = f(b) \in P_{\beta_k}$. It implies that $\beta_k = \alpha$ for some $\alpha \in A_{C,k}$, hence $b \in U_\alpha$. Therefore, $\{U_\alpha : \alpha \in A_{C,k}\}$ is an uncountable open cover for $f^{-1}(C)$, but it has not any proper subcover. So $f^{-1}(C)$ is not separable, hence f is not a *cs*-mapping. It is a contradiction.

Sufficiency. Let $\bigcup\{\mathcal{P}_n^* : n \in \mathbb{N}\}$ be a σ-compact-countable network for X. For each compact subset C of X, we have that $A_{C,n} = \{\alpha \in A_n : C \cap P_\alpha \neq \emptyset\}$ is countable for every $n \in \mathbb{N}$. Then $\prod_{n\in\mathbb{N}} A_{C,n}$ is hereditarily separable. It follows from $f^{-1}(C) \subset \prod_{n\in\mathbb{N}} A_{C,n}$ that $f^{-1}(C)$ is separable. Then f is a *cs*-mapping. $\square$

For a Ponomarev-system $(f, M, X, \{\mathcal{P}_n\})$, the following result is well-known.

Lemma 2.11 ([21], Lemma 2.2)**.** *Let $(f, M, X, \{\mathcal{P}_n\})$ be a Ponomarev-system. If $\bigcup\{\mathcal{P}_n : n \in \mathbb{N}\}$ is a σ-strong network for X, then f is a π-mapping.*

In the next, we give a sufficient condition for f to be a π-mapping in a Σ-Ponomarev-system $(f, M, X, \{\mathcal{P}_n^*\})$.

Proposition 2.12. *Let $(f, M, X, \{\mathcal{P}_n^*\})$ be a Σ-Ponomarev-system. If $\bigcup\{\mathcal{P}_n^* : n \in \mathbb{N}\}$ is a σ-strong network for X, then f is a π-mapping.*

Proof. Let $x \in U$ with U open in X. Since $\bigcup\{\mathcal{P}_n^* : n \in \mathbb{N}\}$ is a σ-strong network for X, there exists $n \in \mathbb{N}$ such that $st(x, \mathcal{P}_n^*) \subset U$. If $a = (\alpha_n) \in M$ such that $d(f^{-1}(x), a) < \dfrac{1}{2^n}$, then there exists $b = (\beta_n) \in f^{-1}(x)$ such that $d(a, b) < \dfrac{1}{2^n}$. It implies that $\alpha_k = \beta_k$ if $k \leq n$. Then $f(a) \in P_{\alpha_n} = P_{\beta_n} \subset st(x, \mathcal{P}_n^*) \subset U$, hence $a \in f^{-1}(U)$. Therefore, $d(f^{-1}(x), M - f^{-1}(U)) \geq \dfrac{1}{2^n} > 0$. This proves that f is a π-mapping. □

The following example shows that the inverse implication of Proposition 2.12 does not hold.

Example 2.13. There exists a Σ-Ponomarev-system $(f, M, X, \{\mathcal{P}_n^*\})$ such that the following holds.

(1) f is a π-mapping.
(2) $\bigcup\{\mathcal{P}_n^* : n \in \mathbb{N}\}$ is not a σ-strong network for X.

Proof. Let $X = \{x, y, z\}$ be a discrete space. Put $\mathcal{P}_1 = \{\{x\}, \{z\}, \{x, y\}\}$, and $\mathcal{P}_n = \{\{x\}, \{y\}, \{z\}, \{x, z\}\}$ for every $n \geq 2$. It is easy to see that $\mathcal{P} = \bigcup\{\mathcal{P}_n : n \in \mathbb{N}\}$ is a σ-network for X, then the Σ-Ponomarev-system $(f, M, X, \{\mathcal{P}_n^*\})$ exists. Note that $\mathcal{P}_n^* = \mathcal{P}_n$ for every $n \in \mathbb{N}$.

(1). f is a π-mapping.

For each $t \in X$ and each neighborhood U of t, since $\{t\}$ is also a neighborhood of t and $\{t\} \subset U$, we have that $d(f^{-1}(t), M - f^{-1}(U)) \geq d(f^{-1}(t), M - f^{-1}(t)) = \inf\{d(a, b) : a \in f^{-1}(t), b \in M - f^{-1}(t)\}$. For each $a = (\alpha_n) \in f^{-1}(t)$ and $b = (\beta_n) \in M - f^{-1}(t)$, we consider two following cases (a) and (b).

(a) $t = x$ or $t = y$. If $\beta_1 \neq \alpha_1$, then $d(a,b) \geq \dfrac{1}{2}d(\alpha_1, \beta_1) = \dfrac{1}{2}$. If $\beta_1 = \alpha_1$, then $P_{\alpha_1} = P_{\beta_1} = \{x, y\}$. Since $b \in M - f^{-1}(t)$, we have that $P_{\beta_2} = \{y\}$ if $t = x$, and $P_{\beta_2} = \{x\}$ if $t = y$. It implies that $\beta_2 \neq \alpha_2$. Then $d(a,b) \geq \dfrac{1}{2^2}d(\alpha_2, \beta_2) = \dfrac{1}{4}$.

(b) $t = z$. We have that $\alpha_1 \neq \beta_1$, then $d(a,b) \geq \dfrac{1}{2}d(\alpha_1, \beta_1) = \dfrac{1}{2}$.

By the above, $d(a,b) \geq \dfrac{1}{4}$ for every $a \in f^{-1}(t)$ and $b \in M - f^{-1}(t)$. Then $d(f^{-1}(t), M - f^{-1}(U)) \geq d(f^{-1}(t), M - f^{-1}(t)) \geq \dfrac{1}{4} > 0$. This proves that f is a π-mapping.

(2). $\bigcup\{\mathcal{P}_n^* : n \in \mathbb{N}\}$ is not a σ-strong network for X.

Let $U = \{x\}$, then U is a neighborhood of x in X. We have that $st(x, \mathcal{P}_1^*) = \{x, y\}$, and $st(x, \mathcal{P}_n^*) = \{x, z\}$ for every $n \geq 2$. It implies that $st(x, \mathcal{P}_n^*) \not\subset U$ for every $n \in \mathbb{N}$. Then $\bigcup\{\mathcal{P}_n^* : n \in \mathbb{N}\}$ is not a σ-strong network for X. □

Remark 2.14. By Remark 2.8.(4), Example 2.13 also shows that the inverse implication of Lemma 2.11 does not hold.

We do not know whether a necessary and sufficient condition for f to be a π-mapping can be obtained in a Σ-Ponomarev-system $(f, M, X, \{\mathcal{P}_n^*\})$. To get a necessary and sufficient condition such that $\bigcup\{\mathcal{P}_n^* : n \in \mathbb{N}\}$ is a σ-strong network for X, we introduce the following notion.

Definition 2.15. Let $f : X \longrightarrow Y$ be a mapping. f is a *complete* mapping if, for each $y \in Y$, $f^{-1}(y)$ is a complete metric subset of X.

Proposition 2.16. *Let $(f, M, X, \{\mathcal{P}_n^*\})$ be a Σ-Ponomarev-system. Then $\bigcup\{\mathcal{P}_n^* : n \in \mathbb{N}\}$ is a σ-strong network for X if and only if f is a complete mapping.*

Proof. Necessity. Let $\bigcup\{\mathcal{P}_n^* : n \in \mathbb{N}\}$ be a σ-strong network for X. For each $x \in X$, put $A_{x,n} = \{\alpha \in A_n : x \in P_\alpha\}$. Then $\{P_{\alpha_n} : n \in \mathbb{N}\}$ forms a network at x in X for every $(\alpha_n) \in \prod_{n\in\mathbb{N}} A_{x,n}$. Therefore, $f^{-1}(x) = \prod_{n\in\mathbb{N}} A_{x,n}$. It follows from [3, Theorem 4.3.12] that $f^{-1}(x)$ is a complete metric subset of M. Then f is a complete mapping.

Sufficiency. Let f be a complete mapping. If $\bigcup\{\mathcal{P}_n^* : n \in \mathbb{N}\}$ is not a σ-strong network for X. By using notations and arguments in proof (a) of the necessity of Theorem 2.10.(2) again, we have that $a \notin f^{-1}(x)$, and the sequence $\{a_i : i \in \mathbb{N}\}$ converges to a in $\prod_{n\in\mathbb{N}} A_n$. Clearly, $\{a_i : i \in \mathbb{N}\}$ is a Cauchy sequence in $f^{-1}(x)$. Since $f^{-1}(x)$ is a complete metric subset of M, $a \in f^{-1}(x)$. It is a contradiction. □

In [6], [7], [8], [9], necessary and sufficient conditions such that f is a covering-mapping have been obtained for Ponomarev-systems $(f, M, X, \mathcal{P})$ and $(f, M, X, \{\mathcal{P}_n\})$. Next, we give necessary and sufficient conditions for f to be a covering-mapping in a Σ-Ponomarev-system $(f, M, X, \{\mathcal{P}_n^*\})$.

Theorem 2.17. *Let $(f, M, X, \{\mathcal{P}_n^*\})$ be a Σ-Ponomarev-system. Then the following hold.*

(1) *f is sequence-covering if and only if $\bigcup\{\mathcal{P}_n^* : n \in \mathbb{N}\}$ is a σ-cs-network for X.*
(2) *f is sequentially-quotient (subsequence-covering) if and only if $\bigcup\{\mathcal{P}_n^* : n \in \mathbb{N}\}$ is a σ-cs*-network for X.*
(3) *f is 1-sequence-covering if and only if $\bigcup\{\mathcal{P}_n^* : n \in \mathbb{N}\}$ is a σ-sn-network for X.*
(4) *f is 2-sequence-covering if and only if $\bigcup\{\mathcal{P}_n^* : n \in \mathbb{N}\}$ is a σ-so-network for X.*

Proof. (1). *Necessity.* Let f be a sequence-covering mapping. Then for each convergent sequence $S = \{x_n : n \in \omega\}$ converging to x_0 in X, there exists a convergent sequence $C = \{a_n : n \in \omega\}$ converging to a_0 in M such that $f(a_n) = x_n$ for every $n \in \omega$. Let $a_0 = (\alpha_{0n}) \in M$. Then $\{P_{\alpha_{0n}} : n \in \mathbb{N}\}$ is a network at x_0 in X. For each $k \in \mathbb{N}$, we have that $U_k = \{b = (\beta_n) \in M : \beta_k = \alpha_{0k}\}$ is a neighborhood of a_0 in M. Then C is eventually in U_k. It implies that S is eventually in $P_{\beta_k} = P_{\alpha_{0k}}$. Then $\bigcup\{\mathcal{P}_n^* : n \in \mathbb{N}\}$ is a σ-*cs*-network for X.

Sufficiency. Let $\bigcup\{\mathcal{P}_n^* : n \in \mathbb{N}\}$ be a σ-*cs*-network for X. For each sequence $S = \{x_m : m \in \omega\}$ converging to x_0 in X, there exists $\{P_{\alpha_{0n}} : n \in \mathbb{N}\} \subset \mathcal{P}$ such that $\{P_{\alpha_{0n}} : n \in \mathbb{N}\}$ is a σ-network at x_0 in X and S is eventually in $P_{\alpha_{0n}}$ for every $n \in \mathbb{N}$. For each $m \in \mathbb{N}$, let $x_m = f(b_m)$ where $b_m = (\beta_{mn}) \in M$. For each $m \in \mathbb{N}$ and $n \in \mathbb{N}$, put $\alpha_{mn} = \alpha_{0n}$ if $x_m \in P_{\alpha_{0n}}$, and $P_{\alpha_{mn}} = \bigcap_{j=1}^{n} P_{\beta_{mj}}$ if $x_m \notin P_{\alpha_{0n}}$. We have that $a_m = (\alpha_{mn}) \in f^{-1}(x_m)$ for every $m \in \omega$.

Since S is eventually in $P_{\alpha_{0n}}$ for every $n \in \mathbb{N}$, there exists $m(n)$ such that $\alpha_{mn} = \alpha_{0n}$ for every $m \geq m(n)$. Then $\{a_m : m \in \omega\}$ converges to a_0 in M. This prove that f is a sequence-covering mapping.

(2). *Necessity.* Let f be a sequentially-quotient mapping. For each convergent sequence $S = \{x_m : m \in \omega\}$ converging to x_0 in X, there exists a convergent sequence C in M such that $f(C)$ is a subsequence of S. By using arguments as in the necessity of (1) again, there exists $\{P_{\alpha_{0n}} : n \in \mathbb{N}\} \subset \mathcal{P}$ such that $\{P_{\alpha_{0n}} : n \in \mathbb{N}\}$ is a σ-network at x_0 in X and $f(C)$ is eventually in $P_{\alpha_{0n}}$ for every $n \in \mathbb{N}$. Then $\bigcup\{\mathcal{P}_n^* : n \in \mathbb{N}\}$ is a σ-cs^*-network for X.

The parenthetic part is obvious by [5, Proposition 2.1].

Sufficiency. Let $\bigcup\{\mathcal{P}_n^* : n \in \mathbb{N}\}$ be a σ-cs^*-network for X. For each sequence $S = \{x_m : m \in \omega\}$ converging to x_0 in X there exists a subsequence L of S and $\{P_{\alpha_{0n}} : n \in \mathbb{N}\} \subset \mathcal{P}$ such that $\{P_{\alpha_{0n}} : n \in \mathbb{N}\}$ is a σ-network at x in X and L is eventually in $P_{\alpha_{0n}}$ for every $n \in \mathbb{N}$. By using arguments as in the sufficiency of (1) again, there exists a convergent sequence C in M such that $f(C) = L$. It implies that f is a sequentially-quotient mapping.

The parenthetic part is obvious by the fact that every sequentially-quotient mapping is a subsequence-covering mapping.

(3). *Necessity.* Let f be an 1-sequence-covering mapping. For each $x \in X$, there exists $a_x \in M$ such that whenever $\{x_n : n \in \mathbb{N}\}$ is a sequence converging to x in X there exists a sequence $\{a_n : n \in \mathbb{N}\}$ converging to a_x in M with each $a_n \in f^{-1}(x_n)$. Put $a_x = (\alpha_n)$, then $\{P_{\alpha_n} : n \in \mathbb{N}\}$ is a σ-network at x in X.

If $\{y_n : n \in \mathbb{N}\}$ is a sequence converging to x, there exists a sequence $\{b_n : n \in \mathbb{N}\}$ converging to a_x in M with each $b_n \in f^{-1}(y_n)$. For each $k \in \mathbb{N}$, put $U_k = \{b = (\beta_n) \in M : \beta_k = \alpha_k\}$. Then U_k is a neighborhood of a_x in M, hence $\{b_n : n \in \mathbb{N}\} \cup \{a_x\}$ is eventually in U_k. By Lemma 2.9, $\{y_n : n \in \mathbb{N}\} \cup \{x\}$ is eventually in $f(U_k) = \bigcap_{n=1}^{k} P_{\alpha_n} \subset P_{\alpha_k}$. This proves that P_{α_k} is a sequential neighborhood of x in X.

By the above, $\bigcup\{\mathcal{P}_n^* : n \in \mathbb{N}\}$ is a σ-sn-network for X.

Sufficiency. Let $\bigcup\{\mathcal{P}_n^* : n \in \mathbb{N}\}$ be a σ-sn-network for X. For each $x \in X$ there exists $\{P_{\alpha_n} : n \in \mathbb{N}\} \subset \mathcal{P}$ such that $\{P_{\alpha_n} : n \in \mathbb{N}\}$ is a σ-network at x in X and each P_{α_n} is a sequential neighborhood of x in X. Put $a_x = (\alpha_n)$, then $a_x \in f^{-1}(x)$. For each $n \in \mathbb{N}$, if $\{x_m : m \in \mathbb{N}\}$ is a sequence converging to x in X, then $\{x_m : m \in \mathbb{N}\} \cup \{x\}$ is eventually in P_{α_n}. For each $m \in \mathbb{N}$, let $x_m = f(b_m)$ where $b_m = (\beta_{mn}) \in M$. For each $m \in \mathbb{N}$ and $n \in \mathbb{N}$, put $P_{\alpha_{mn}} = P_{\alpha_n}$ if $x_m \in P_{\alpha_n}$, and $P_{\alpha_{mn}} = \bigcap_{j=1}^{n} P_{\beta_{mj}}$ if $x_m \notin P_{\alpha_n}$. Then $a_m = (\alpha_{mn}) \in f^{-1}(x_m)$ for every $m \in \mathbb{N}$. Since S is eventually in P_{α_n} for every $n \in \mathbb{N}$, there exists $m(n)$ such that $\alpha_{mn} = \alpha_n$ for every $m \geq m(n)$. Then $\{a_m : m \in \mathbb{N}\}$ converges to a_x in M. This prove that f is an 1-sequence-covering mapping.

(4). *Necessity.* Let f be a 2-sequence-covering mapping. For each $x \in X$, there exists $a_x = (\alpha_n) \in M$ such that $f(a_x) = x$. Then $\{P_{\alpha_n} : n \in \mathbb{N}\} \subset \mathcal{P}$ is a σ-network at x in X.

If $\{P_{\alpha_n} : n \in \mathbb{N}\} \subset \mathcal{P}$ is a σ-network at x in X. Put $a_x = (\alpha_n)$, then $a_x \in f^{-1}(x)$. If $\{x_n : n \in \mathbb{N}\}$ is a sequence converging to x, there exists a sequence $\{a_n : n \in \mathbb{N}\}$ converging to a_x in M with each $a_n \in f^{-1}(x_n)$. As in the necessity of (3), P_{α_k} is a sequential neighborhood of x in X for every $k \in \mathbb{N}$.

By the above, $\bigcup\{\mathcal{P}_n^* : n \in \mathbb{N}\}$ is a σ-*so*-network for X.

Sufficiency. Let $\mathcal{P}$ be a σ-*so*-network for X. For each $x \in X$ and $a_x = (\alpha_n) \in f^{-1}(x)$, $\{P_{\alpha_n} : n \in \mathbb{N}\}$ is a σ-network at x in X and each P_{α_n} is a sequential neighborhood of x in X. As in the sufficiency of (3), there exists a sequence $\{a_m : m \in \mathbb{N}\}$ converging to a_x in M with each $a_m \in f^{-1}(x_m)$. This proves that f is a 2-sequence-covering mapping. □

By Theorem 2.10, Proposition 2.16, and Theorem 2.17, we get following corollaries.

Corollary 2.18. *Let $(f, M, X, \{\mathcal{P}_n^*\})$ be a Σ-Ponomarev-system. Then the following are equivalent, where "sequence-covering" and "cs-" can be replaced by "1-sequence-covering" and "sn-" ("2-sequence-covering" and "so-", "sequentially-quotient" and "cs*-") respectively; and "s-mapping" and "point-countable" can be replaced by "msss-mapping" and "locally countable" ("mssc-mapping" and "locally finite", "cs-mapping" and "compact-countable") respectively.*

(1) *f is a sequence-covering s-mapping.*
(2) *$\bigcup\{\mathcal{P}_n^* : n \in \mathbb{N}\}$ is a σ-point-countable cs-network for X.*

Corollary 2.19. *Let $(f, M, X, \{\mathcal{P}_n^*\})$ be a Σ-Ponomarev-system. Then the following are equivalent, where "sequence-covering" and "cs-" can be replaced by "1-sequence-covering" and "sn-" ("2-sequence-covering" and "so-", "sequentially-quotient" and "cs*-") respectively.*

(1) *f is a sequence-covering compact mapping.*
(2) *$\bigcup\{\mathcal{P}_n^* : n \in \mathbb{N}\}$ is a σ-point-finite strong cs-network for X.*

Corollary 2.20. *Let $(f, M, X, \{\mathcal{P}_n^*\})$ be a Σ-Ponomarev-system. Then the following are equivalent, where "sequence-covering" and "cs-" can be replaced by "1-sequence-covering" and "sn-" ("2-sequence-covering" and "so-", "sequentially-quotient" and "cs*-") respectively.*

(1) *f is a sequence-covering complete mapping.*
(2) *$\bigcup\{\mathcal{P}_n^* : n \in \mathbb{N}\}$ is a σ-strong cs-network for X.*

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References

[1] A. V. Arhangel'skii, *Mappings and spaces*, Russian Math. Surveys **21** (1966), 115–162.

[2] J. R. Boone and F. Siwiec, *Sequentially quotient mappings*, Czechoslovak Math. J. **26** (1976), 174–182.

[3] R. Engelking, *General topology*, Sigma series in pure mathematics, vol. 6, Heldermann Verlag, Berlin, 1988.

[4] S. P. Franklin, *Spaces in which sequences suffice*, Fund. Math. **57** (1965), 107–115.

[5] Y. Ge, *On compact images of locally separable metric spaces*, Topology Proc. **27** (2003), no. 1, 351–360.

[6] Y. Ge, *Mappings in Ponomarev-systems*, Topology Proc. **29** (2005), no. 1, 141–153.

[7] Y. Ge, *On three equivelances concerning Ponomarev-systems*, Arch. Math. (Brno) **42** (2006), 239–246.

[8] Y. Ge and S. Lin, *On Ponomarev-systems*, Boll. Unione Mat. Ital. Sez. B Artic. Ric. Mat. (8) **10** (2007), 455–467.

[9] Y. Ge and J. Shen, *Networks in Ponomarev-systems*, Sci. Ser. A Math. Sci. **11** (2005), 25–29.

[10] Y. Ikeda, C. Liu, and Y. Tanaka, *Quotient compact images of metric spaces, and related matters*, Topology Appl. **122** (2002), 237– 252.

[11] Z. Li, Q. Li, and X. Zhou, *On sequence-covering msss-maps*, Mat. Vesnik **59** (2007), 15–21.

[12] S. Lin, *Locally countable families, locally finite families and Alexandroff's problem*, Acta Math. Sinica (Chin. Ser.) **37** (1994), 491–496.

[13] S. Lin, *On sequence-covering s-mappings*, Adv. Math. (China) **25** (1996), 548–551.

[14] S. Lin, *Point-countable covers and sequence-covering mappings*, Chinese Science Press, Beijing, 2002.

[15] S. Lin, C. Liu, and M. Dai, *Images on locally separable metric spaces*, Acta Math. Sinica (N.S.) **13** (1997), no. 1, 1–8.

[16] S. Lin and P. Yan, *Notes on cfp-covers*, Comment. Math. Univ. Carolin. **44** (2003), no. 2, 295 –306.

[17] E. Michael, $\aleph_0$*-spaces*, J. Math. Mech. **15** (1966), 983– 1002.

[18] Z. Qu and Z. Gao, *Spaces with compact-countable k-networks*, Math. Japonica **49** (1999), 199 –205.

[19] F. Siwiec, *Sequence-covering and countably bi-quotient mappings*, General Topology Appl. **1** (1971), 143–154.

[20] Y. Tanaka, *Theory of k-networks II*, Questions Answers in Gen. Topology **19** (2001), 27–46.

[21] Y. Tanaka and Y. Ge, *Around quotient compact images of metric spaces, and symmetric spaces*, Houston J. Math. **32** (2006), no. 1, 99–117.

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