TOPOLOGY PROCEEDINGS

Volume 36, 2010

Pages 131–140

http://topology.auburn.edu/tp/

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Electronically published on February 12, 2010

Topology Proceedings

Web:	http://topology.auburn.edu/tp/
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E-mail:	topolog@auburn.edu
ISSN:	0146-4124

http://topology.auburn.edu/tp/

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Volume 36 (2010)
Pages 131-140

E-Published on February 12, 2010

A NON-METRIC SEPARABLE CONTINUUM WHOSE HYPERSPACE OF SUBCONTINUA IS NOT SEPARABLE

MICHEL SMITH

Abstract. We present a technique for producing separable non-metric continua whose hyperspace of subcontinua is not separable. Two specific examples are constructed: 1. a $\sin(\frac{1}{x})$ type of continuum with a metric ray limiting on a nonmetric portion; 2. a metric Cantor tree limiting on a non-metric space. In examining the non-metrizability of the construction, the following theorem was proven: If X is a connected, locally compact, second countable metric space and $\gamma(X)$ is a compactification of X so that the remainder $\gamma(X) - X$ is totally disconnected, then $\gamma(X)$ is metric.

1. Introduction

This work is an offshoot of the consideration of non-metric continua whose hyperspace of subcontinua support a Whitney map. In [1], Janusz J. Charatonik and Wlodzimierz J. Charatonik give an example of such a continuum based on an example of Andrzej Gutek and Charles L. Hagopian [4]. Jennifer Stone in her dissertation [6] (also [7]) developed a technique using inverse limits to produce "hereditarily nonmetric" continua that support Whitney maps. These are continua every proper subcontinuum of which is non-metric. This author was interested in finding a different class of non-metric continua that support Whitney maps. It was observed

2010 *Mathematics Subject Classification.* Primary 54F15, 54D35; Secondary 54B20.

Key words and phrases. compactifications, hyperspace of subcontinua, non-metric continua.

that a potential technique that could be used to produce such a continua would be to build a metric Cantor tree that limits to a compact non-metric space in such a way that the Whitney map on the metric tree can be extended to its compactification. In particular, if the remainder happens to be totally disconnected, such as the double arrow space, and does not have any "loops," then the Whitney map on the metric portion should be easily extendable. One of the results of this paper argues that such a plan is impossible, since such a continuum turns out to be metric. One of the planned examples, which does not support a Whitney map (this follows from results in [6]), had the interesting property of being a separable continuum whose hyperspace of subcontinua is not separable.

2. Main results

Let I denote the unit interval $[0,1]$. Let $I_t = [0,1]$ for $t \in I$.

Let $X = \Pi_{t\in I} I_t$. Then X is a compact non-metric Hausdorff continuum. Furthermore, X may be thought of as the set of all functions from $[0,1]$ into $[0,1]$. It is known that X is separable. Let X^+ denote $X \times I$. For $P \in X$ and $x \in I$, let $P(x)$ denote the x^{th} coordinate of P.

Suppose that $t_1, t_2, ..., t_n$ is a sequence of elements of I and $O_1, O_2, ..., O_n$ is a sequence of open sets in I. We define the basic open sets corresponding to these choices: $R[t_1, ...t_n](O_1, O_2, ..., O_n)$ denotes the basic open set $\{P \in X | P(t_i) \in O_i \forall i\}$.

For each $x, y \in [0,1]$, let $J_x(y)$ denote the point of X defined as follows:

$$\text{let } J_x(y)(t) = \begin{cases} 0 & \text{if } t < x \\ y & \text{if } t = x \\ 1 & \text{if } t > x. \end{cases}$$

Let $L = \{J_x(y) | x, y \in [0,1]\}$.

Observation. L is homeomorphic to the standard lexicographically ordered square.

Example 1. A metric ray limiting to the lexicographic arc whose hyperspace of subcontinua is not separable.

Construction of Example 1. For $0 < i \leq 2^n$, let $\alpha_{i,n}$ be defined as follows: for $t \in [0,1]$, $\alpha_{i,n}[t]$ is the following point in X,

$$\text{let } \alpha_{i,n}[t](x) = \begin{cases} 0 & \text{if } x < \frac{i-1}{2^n} \\ t & \text{if } \frac{i-1}{2^n} \leq x < \frac{i}{2^n} \\ 1 & \text{if } \frac{i}{2^n} \leq x. \end{cases}$$

For $i = 2^n$,

$$\text{let } \alpha_{i,n}[t](x) = \begin{cases} 0 & \text{if } x < \frac{2^n-1}{2^n} \\ t & \text{if } \frac{2^n-1}{2^n} \leq x \leq 1 \ . \end{cases}$$

We note that $\alpha_{i,n} : [0,1] \to X$ is a continuous map, so we have the following claim.

CLAIM 1. The set $\alpha_{i,n} = \{\alpha_{i,n}[t] \ t \in [0,1]\}$ is a metric arc.

By construction it is easy to verify the following claim.

CLAIM 2. If $h \in \alpha_{i,n}$, then h is a nondecreasing function.

If $x_1 \neq x_2$ are such that $h(x_1) \notin \{0,1\}$ and $h(x_2) \notin \{0,1\}$, then $|x_1 - x_2| \leq \frac{1}{2^n}$.

CLAIM 3. The arc $\alpha_{i,n}$ connects two points of L: the point $J_{\frac{i-1}{2^n}}(1) = \alpha_{i,n}[1]$ to the point $J_{\frac{i-1}{2^n}}(0) = \alpha_{i,n}[0]$.

Define the following γ arcs in $X \times I$:

$\gamma_n = (\cup_{i=1}^{2^n} \alpha_{i,n}) \times \{\frac{1}{n}\}$.

CLAIM 4. The arc γ_n connects the point $\{J_0(0)\} \times \{\frac{1}{n}\}$ to the point $\{J_1(1)\} \times \{\frac{1}{n}\}$.

Define the following β arcs in $X \times I$:

For n even $\beta_n = \{J_0(0)\} \times [\frac{1}{n}, \frac{1}{n+1}]$;

For n odd $\beta_n = \{J_1(1)\} \times [\frac{1}{n}, \frac{1}{n+1}]$.

Note that β_n connects an endpoint of γ_n to an endpoint of γ_{n+1}.

CLAIM 5. $\overline{\left(\bigcup_{n=1}^{\infty} \beta_n \cup \gamma_n\right)} = \left(\bigcup_{n=1}^{\infty} \beta_n \cup \gamma_n\right) \bigcup \left(L \times \{0\}\right)$.

Proof of Claim 5: Let Y denote the left side of the equation and Y' the right side.

For $\epsilon > 0$, note that $Y' \cap (X \times [\epsilon, 1])$ is a finite union of metric arcs and so the only limit points of Y' not in Y' must lie in $X \times \{0\}$. So what remains to be shown is (i) that every point of $L \times \{0\}$ is a limit point of $\left(\bigcup_{n=1}^{\infty} \beta_n \cup \gamma_n\right)$, and (ii) that there are no additional limit points.

Let $J_x(y) \in L$ and suppose that $t_1 < t_2 < ... < t_j, ... < t_N$ is a sequence of elements of I and $O_1, O_2, ..., O_N$ is a sequence of basic open sets in I so that if $R = R[t_1, ...t_N](O_1, O_2, ..., O_N)$, then $R \times [0, \epsilon)$ is a basic open set containing $J_x(y) \times \{0\}$; and assume that $t_j = x$. Since $J_x(y) \in R$, we must have

$$0 \in O_i \text{ for } i < j, \qquad y \in O_j, \qquad 1 \in O_i \text{ for } j < i.$$

Select n large enough so that

$$\tfrac{1}{2^n} < \max\{|t_i - t_k|\}_{i \neq k} \qquad \text{and} \qquad \tfrac{1}{n} < \epsilon.$$

Then there exists an integer m so that $\frac{m-1}{2^n} \leq t_j < \frac{m}{2^n}$. Then the arc $\alpha_{m,n}$ intersects R and since $\frac{1}{n} < \epsilon$, the arc γ_n intersects $R \times [0, \epsilon)$. This proves (i).

Suppose now that there is a limit point P of Y' not in $L \times \{0\}$. Then P must lie in $X \times \{0\}$. So let $P = (h, 0)$ and assume $h \notin L$.

Case 1: There exist two indices x_1 and x_2 so that $h(x_1) \notin \{0, 1\}$ and $h(x_2) \notin \{0, 1\}$. Then select n so that $\frac{1}{2^n} < |x_1 - x_2|$ and $\epsilon < \frac{1}{n+1}$. Let $R = R[x_1, x_2]((\frac{h(x_1)}{2}, \frac{h(x_1)+1}{2}), (\frac{h(x_2)}{2}, \frac{h(x_2)+1}{2}))$. Then by Claim 2, for $k > n$, no $\alpha_{i,k}$ intersects R. Also for $k \leq n$, the arcs β_k and γ_k lie in $X \times (\epsilon, 1]$. Thus, $R \times [0, \epsilon)$ misses all the β and γ arcs and so P is not a limit point of Y'.

Case 2: There is only one $x_1 \in [0, 1]$ so that $h(x_1) \notin \{0, 1\}$ and there is an x_2 so that either $x_1 < x_2$ and $h(x_2) = 0$ or $x_2 < x_1$ and $h(x_2) = 1$. The argument for Case 1 applies here except for the slight modification of the open sets R to address the fact that when $h(x_2) = 0$ or $h(x_2) = 1$ the open sets contain their endpoints.

Case 3: There exist three points $x_1 < x_2 < x_3$ so that either $h(x_1) = 1, h(x_2) = 0, h(x_3) = 1$ or $h(x_1) = 0, h(x_2) = 1, h(x_3) = 0$. In this case it is easy to find an open set in R containing h so that no non-decreasing element of X lies in R.

The proof of Claim 5 is complete. □

For ease of notation, identify the subset $L \times \{0\}$ of Y with L.

Claim 6. The space Y is separable.

Proof of Claim 6: The subspace Y is the closure of the union of countably many metric arcs and so must be separable. □

Let Z be the decomposition space obtained from Y as follows. For each x, $J_x = \{J_x(y) | y \in [0, 1]\}$ is one of the maximal metric intervals of L. For each x and $r \in [0, \frac{1}{4}]$, identify $J_x(\frac{1}{4}+r)$ with $J_x(\frac{3}{4}-r)$. Then if $\mathcal{G}$ is the collection of all pairs $\{J_x(\frac{1}{4}+r), J_x(\frac{3}{4}-r)\}$ of points of L for $x \in [0,1], r \in [0, \frac{1}{4}]$ and singletons for the remainder of points of Y. Then $\mathcal{G}$ is an upper semi-continuous decomposition of Y. The decomposition space $Z = Y/\mathcal{G}$ is a continuum.

In the following, for ease of discussion, the author may blur the distinction between points and sets in the space Y and the decomposition space $Z = Y/\mathcal{G}$ especially where the elements of $\mathcal{G}$ under consideration are singletons or when no confusion should arise.

Notation. If $J \subset Y$, then let $J/\mathcal{G}$ denote the subset of Z consisting of all the elements of $\mathcal{G}$ that lie in J.

Claim 7. The hyperspace $C(Z)$ of subcontinua of Z is not separable.

Proof of Claim 7: Suppose that $R_1, R_2, ..., R_n$ is a sequence of open sets in Z. Then a basis element in $C(Z)$ is $O(R_1, R_2, ..., R_n) = \{M \in C(Z) | M \subset \cup_{i=1}^n R_i, M \cap R_i \neq \emptyset \forall i\}$.

Note that for each x in $\overline{I}$, $J_x/\mathcal{G}$ is a topological T. Let $M_x = \{J_x(y)|\frac{1}{16} \leq y < \frac{1}{4}\} \cup \{J_x(y)|\frac{3}{4} < y < \frac{15}{16}\} \cup \{J_x(\frac{1}{4}), J_x(\frac{3}{4})\}$, then M_x is an arc lying in the "horizontal bar" of the T. Let $\epsilon > 0$. For each x we define three open sets in the decomposition space. Let

$$R_1^x = R[x](\tfrac{1}{32}, \tfrac{4}{32}) \times [0, \epsilon) = \{(h,t) \in Z | \tfrac{1}{32} < h(x) < \tfrac{4}{32}, t < \epsilon\},$$

$$R_2^x = \{(h,t) \in Z | \tfrac{3}{32} < h(x) < \tfrac{9}{32}, t < \epsilon\} \cup \{(\{J_x(\tfrac{1}{4}+r), J_x(\tfrac{3}{4}-r)\}, t) | r < \tfrac{1}{32}, t < \epsilon\} \cup \{(h,t) \in Z | \tfrac{23}{32} < h(x) < \tfrac{29}{32}, t < \epsilon\},$$

$$R_3^x = R[x](\tfrac{28}{32}, \tfrac{31}{32}) \times [0, \epsilon) = \{(h,t) \in Z | \tfrac{28}{32} < h(x) < \tfrac{31}{32}, t < \epsilon\}.$$

Then let $O^x = O(R_1^x, R_2^x, R_3^x)$ be the indicated open set in $C(Z)$.

Subclaim 7.1. O^x contains the point M_x of $C(Z)$.

Proof of Subclaim 7.1: This follows from the construction of O^x.

Subclaim 7.2. No subcontinuum of $\beta_n/\mathcal{G}$ lies in O^p and no subcontinuum of $\gamma_n/\mathcal{G}$ lies in O^p for all $p \in [0,1]$.

Proof of Subclaim 7.2: Such a continuum would be an arc A. Note that all the points of A except the endpoints are singletons in $\mathcal{G}$. In order for A to lie in O^p it must be the case that $\pi_p(A)$ intersect both open segments $(\frac{1}{32}, \frac{4}{32})$ and $(\frac{28}{32}, \frac{31}{32})$ but not contain any points of the interval $[\frac{9}{32}, \frac{23}{32}]$ which is impossible by the connectedness of A.

SUBCLAIM 7.3. $O^p \cap O^q = \emptyset$ if $p \neq q$.

Proof of Subclaim 7.3: In order for a continuum K to be in O^p it must be a subset of $L/\mathcal{G}$ so that $\pi_p(K)$ is a subset of the open segment $(\frac{1}{32}, \frac{31}{32})$. Furthermore, $\pi_x(K) = 0$ for $x < p$ and $\pi_x(K) = 1$ for $p < x$.

Therefore, $C(Z)$ has an uncountable collection of disjoint open sets and so cannot be separable. This demonstrates that the example is not separable, and the proof of Claim 7 is complete. □

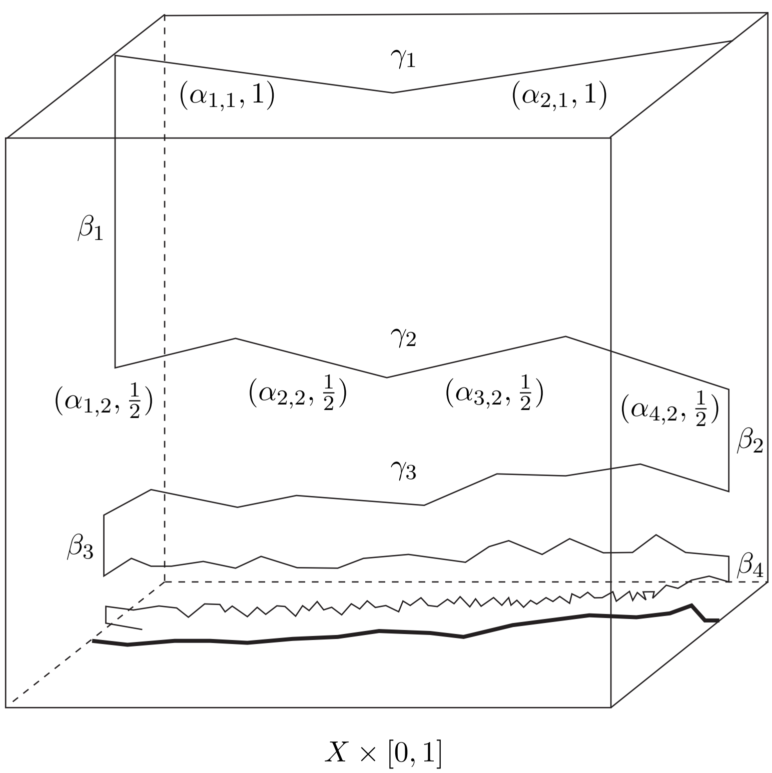

FIGURE 1. Example 1

Example 2. The metric Cantor tree limiting to the lexicographic arc whose hyperspace of subcontinua is not separable.

Note that modification to the interval can produce different examples. In the above and following examples, we used "T's" but there are a wealth of different continua that can be used to serve the purpose. (The referee indicated that different techniques can also be used, and he called our attention to [2], a paper by D. Daniel, J. Nikiel, L. B. Treybig, H. M. Tuncali, and E. D. Tymchatyn where they use an inverse limit construction to "explode" each point in the limit bar of the metric $sin(\frac{1}{x})$ continuum.)

Construction of Example 2. The construction is similar to the one for Example 1 and so some of the arguments will be abbreviated.

Let $\{[r_{i,n}, s_{i,n}]\}_{i=1}^{2^n}$ be the 2^n intervals in $[0,1]$ that remain after removal of the open middle thirds intervals at the nth step of the construction of the Cantor set C. Thus, $C = \cap_{n=1}^{\infty}(\cup_{i=1}^{2^n}[r_{i,n}, s_{i,n}])$. Thus, the middle third $(s_{2i-1,n+1}, r_{2i,n+1})$ has been removed from $[r_{i,n}, s_{i,n}]$. Note that $r_{i,n} = r_{2i-1,n+1}$ and $s_{i,n} = s_{2i,n+1}$.

For $0 < i \leq 2^n$, let $\alpha_{i,n}$ be defined as follows:
for $t \in [0,1]$ and $0 < i < 2^{n-1}$, $\alpha_{i,n}[t]$ is the following point in X,

$$\text{let } \alpha_{i,n}[t](x) = \begin{cases} 0 & \text{if } x < r_{i,n} \\ t & \text{if } r_{i,n} \leq x < s_{i,n} \\ 1 & \text{if } s_{i,n} \leq x. \end{cases}$$

For $i = 2^n$,

$$\text{let } \alpha_{i,n}[t](x) = \begin{cases} 0 & \text{if } x < r_{2^n,n} \\ t & \text{if } r_{2^n,n} \leq x \leq 1 \ . \end{cases}$$

As above, we have the following claims.

Claim 1. The set $\alpha_{i,n} = \{\alpha_{i,n}[t] \ t \in [0,1]\}$ is a metric arc.

Claim 2. If $h \in \alpha_{i,n}$, then h is a nondecreasing function. If $x_1 \neq x_2$ are such that $h(x_1) \notin \{0,1\}$ and $h(x_2) \notin \{0,1\}$, then $|x_1 - x_2| \leq \frac{1}{3^n}$.

By construction we have the following claim.

Claim 3. The $\alpha_{i,n}[1] = \alpha_{2i-1,n+1}[1]$ and $\alpha_{i,n}[0] = \alpha_{2i,n+1}[0]$. Furthermore, for a fixed n, the arcs $\{\alpha_{i,n}\}_{i=1}^{2^n}$ are disjoint.

Define the following β arcs in $X \times I$.

$\beta_{2i-1,n+1}[u] = (\alpha_{i,n}[1], u)$ with $u \in [\frac{1}{n}, \frac{1}{n+1}]$. Note that $\beta[\frac{1}{n+1}] = (\alpha_{2i-1,n+1}[1], \frac{1}{n+1})$.

$\beta_{2i,n+1}[u] = (h(n(n+1)(u - \frac{1}{n+1})), u)$ where

$$h[t] = \begin{cases} 0 & \text{if } t < r_{2i-1,n+1} \\ t & \text{if } r_{2i-1,n+1} \leq t < r_{2i,n+1} \\ 1 & \text{if } r_{2i,n+1} \leq t. \end{cases}$$

This is set up so that $h(\frac{1}{n}) = \alpha_{i,n}[1]$ and $h(\frac{1}{n+1}) = \alpha_{2i,n+1}[1]$.

Thus, $\beta_{2i-1,n+1}$ connects $\beta_{i,n}[\frac{1}{n}]$ to $\beta_{2i-1,n+1}[\frac{1}{n+1}]$ and $\beta_{2i,n+1}$ connects $\beta_{i,n}[\frac{1}{n}]$ to $\beta_{2i,n+1}[\frac{1}{n+1}]$. So the union of all the β arcs is a Cantor tree with ramification points $\{\beta_{i,n}[1]\}_{i=1,n=1}^{2^n,\infty}$.

Claim 4. $\overline{\left(\bigcup_{i=1,n=1}^{2^n,\infty} \beta_{i,n}\right)} - \bigcup_{i=1,n=1}^{2^n,\infty} \beta_{i,n} \subset L \times \{0\}$.

Claim 5. The space Y is separable.

The proofs of claims 4 and 5 are sufficiently similar to the proofs of claims 5 and 6 in Example 1 that they are omitted.

As above, let Z be the decomposition space obtained from Y as follows. As above for each x, J_x is one of the maximal metric intervals of L. For each x and $r \in [0, \frac{1}{4}]$, identify $J_x(\frac{1}{4} + r)$ with $J_x(\frac{3}{4} - r)$. Let $\mathcal{G}$ be the collection of all pairs $\{J_x(\frac{1}{4} + r), J_x(\frac{3}{4} - r)\}$ of points of L for $x \in [0,1], r \in [0, \frac{1}{4}]$ and singletons for the remainder of points of Y. Then $\mathcal{G}$ is an upper semi-continuous decomposition of Y. The decomposition space $Z = Y/\mathcal{G}$ is a continuum.

Claim 6. The hyperspace $C(Z)$ of a subcontinua of Z is not separable.

If each of the components of

$$\overline{\left(\bigcup_{i=1,n=1}^{2^n,\infty} \beta_{i,n}\right)} - \bigcup_{i=1,n=1}^{2^n,\infty} \beta_{i,n}$$

from Claim 4 of Example 2 is shrunk to a point, then we have a compactification of the Cantor tree whose remainder is totally disconnected. What can be said about this remainder? Since the double arrow space is a totally disconnected non-metric subset of the lexicographic arc, is there any hope that the Cantor tree can limit down to this set? We now show that our plan to have the Cantor tree limit down to the double arrow space is impossible.

Theorem 3. *Suppose that X is a connected, locally compact, second countable metric space and that $\gamma(X)$ is a compactification of X so that the remainder $\gamma(X) - X$ is totally disconnected. Then $\gamma(X)$ is metric.*

Proof: Let $W = \gamma(X) - X$. We show that $\gamma(X)$ has a countable basis. Let $\mathcal{B}$ be a countable basis for X. Let $\mathcal{G}$ be the set of all pairs (g, h) of finite subcollections of $\mathcal{B}$. Then $\mathcal{G}$ is countable. For each element (g, h) of $\mathcal{G}$, let $U(g, h)$ denote the set to which p belongs if and only if there is a component of $\gamma(X) - \cup g$ intersecting $\overline{\cup h}$ that contains p. Let $\mathcal{H} = \{Int(U(g, h))|(g, h) \in \mathcal{G}\}$.

Note that since X is connected, $\gamma(X)$ is a continuum. We claim that $\mathcal{H}$ forms a basis for W. Let $w \in W$ and let O be an open set in $\gamma(X)$ containing w. Then there is an open set O' containing w so that $\overline{O'} \subset O$ and $Bd(O') \cap W = \emptyset$. Let $K = Bd(O')$. Then K is a compact subset of X and $K \subset O$. So there is finite subcollection g of $\mathcal{B}$ covering K, the closure of each element of which is a subset of O that misses W. Thus, $\overline{\cup g} \subset O$. Then $O' - \overline{\cup g}$ is an open set containing w so there is an open set O'' containing w so that $\overline{O''} \subset O' - \overline{\cup g}$ and $Bd(O'') \cap W = \emptyset$. Let $L = Bd(O'')$; then L is compact. So there is finite subcollection h of $\mathcal{B}$ covering L, the closure of each element of which is a subset of O' that misses w.

For each point $x \in O''$, let C_x be the component of $\overline{O''}$ that contains x. Then, since $\gamma(X)$ is a continuum, $C_x \cap Bd(O'') \neq \emptyset$; thus, $C_x \cap L \neq \emptyset$ and hence, $C_x \cap \cup h \neq \emptyset$. For each $x \in O''$, let D_x be the component of $\gamma(X) - \cup g$ that contains x. Since $O'' \subset O' - \cup g$, we have $C_x \subset D_x$, and so D_x is a component of $\gamma(X) - \cup g$ that intersects $\cup h$. Thus, for each $x \in O''$, $C_x \subset U(g, h)$. Since D_x intersects O' and no point of the boundary of O', it is a subset of O'. Therefore, $O'' \subset Int(U(g, h))$. Now, $\cup h \subset O'$ and g covers $Bd(O')$ so every component of $\gamma(X) - \cup g$ that intersects $\cup h$ must be a subset of O'. Thus, $Int(U(g, h)) \subset O' \subset O$. Clearly, $w \in Int(g, h)$, so this is an open set containing w and lying in O. Therefore, $\mathcal{H}$ is a basis for W. Then $\mathcal{B} \cup \mathcal{H}$ is a countable basis for $\gamma(X)$ and hence, $\gamma(X)$ is metric. □

Since a locally compact connected metric space is completely separable we have the following corollary.

Corollary 4. *Suppose that X is a connected, locally compact, second countable metric space and that $\gamma(X)$ is a compactification of X so that the remainder $\gamma(X) - X$ is totally disconnected. Then $\gamma(X)$ is metric.*

The compactification of the space X described in the theorem is a Freudenthal compactification. It should be noted that J. R. Isbell's development of Freudenthal compactifications in *Uniform Spaces* includes results with similar conclusions and techniques. (See, in particular, [5, VII #42, p. 116].) [1]

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[1] The referee was kind enough to point out this information and the relevant reference to the author.