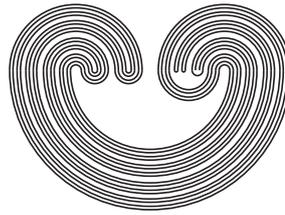


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## METRIC AXIOMS: A STRUCTURAL STUDY

by

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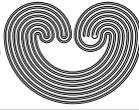
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## METRIC AXIOMS: A STRUCTURAL STUDY

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**ABSTRACT.** For a fixed set  $X$ , an arbitrary *weight structure*  $d \in [0, \infty]^{X \times X}$  can be interpreted as a distance assignment between pairs of points on  $X$ . Restrictions (i.e., *metric axioms*) on the behaviour of any such  $d$  naturally arise, such as separation, triangle inequality and symmetry. We present an order-theoretic investigation of various collections of weight structures, as naturally occurring subsets of  $[0, \infty]^{X \times X}$  satisfying certain metric axioms. Furthermore, we exploit the categorical notion of adjunctions when investigating connections between the above collections of weight structures. As a corollary, we present several lattice-embeddability theorems on a well-known collection of weight structures on  $X$ .

### 1. INTRODUCTION

For a fixed set  $X$ , a **standard metric** on it is any  $d \in [0, \infty]^{X \times X}$  for which:

- (i)  $\forall x \in X, d(x, x) = 0$
- (ii)  $\forall x, y \in X, d(x, y) = 0 = d(y, x) \Rightarrow x = y$  (*separation*)
- (iii)  $\forall x, y \in X, d(x, y) = d(y, x)$  (*symmetry*)
- (iv)  $\forall x, y, z \in X, d(x, y) + d(y, z) \geq d(x, z)$  (*triangle inequality*).

The collection of all standard metrics on  $X$  can then be identified with a particular subset of  $[0, \infty]^{X \times X}$ . For convenience, we shall refer to axioms (i), (ii), (iii) and (iv) as  $0, s, \Sigma$  and  $\Delta$  respectively. By letting  $P$

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*Key words and phrases.* Lattice embedability, metric axioms, metrics spaces, metrizable, Menger convex, definability, adjoints, symmetrization, weight structure, topology, lattice theory, metric dual, premetric, pseudo metric, quasi metric.

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