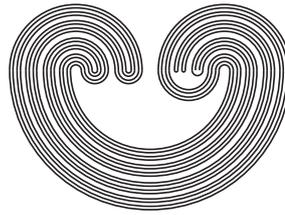


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A FIXED POINT THEOREM FOR CONTRACTING MAPS OF SYMMETRIC CONTINUITY SPACES

by

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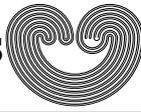
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A FIXED POINT THEOREM FOR CONTRACTING MAPS OF SYMMETRIC CONTINUITY SPACES

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ABSTRACT. We prove a generalization of the Banach fixed point theorem for symmetric separated \mathbb{V} -continuity spaces. We also give examples to show that in general we cannot weaken our assumptions.

1. INTRODUCTION

One of the most important fixed point theorems to arise from the study of metric spaces is the Banach fixed point theorem. This theorem can be stated as follows:

Theorem 1.1 (Banach Fixed Point Theorem). *If (M, d_M) is a Cauchy complete metric space, $m < n \in \mathbb{N}$ and $f : M \rightarrow M$ is a function such that*

$$(\forall x, y \in M) \quad n \cdot d_M(f(x), f(y)) \leq m \cdot d_M(x, y)$$

then f has a unique fixed point.

It is natural to ask, “On what spaces, other than Cauchy complete metric spaces, and for which classes of maps, can such a fixed point theorem be proved?” In [3] it was shown that for every topological space $(T, \mathcal{O}(T))$ there is a quantale \mathbb{V} and \mathbb{V} -continuity space (T, d_T) such that $(T, \mathcal{O}(T))$ is homeomorphic to the topological space of open balls of (T, d_T) . This allows us to refine the above question to “For what quantale’s \mathbb{V} , what \mathbb{V} -continuity spaces (M, d_M) and what class of non-expanding maps does a version of the Banach fixed point theorem hold?”

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