http://topology.auburn.edu/tp/

TOPOLOGY PROCEEDINGS

Volume 47, 2016

Pages 89–100

http://topology.nipissingu.ca/tp/

A Fixed Point Theorem for Contracting Maps of Symmetric Continuity Spaces

by

Nathanael Leedom Ackerman

Electronically published on May 2, 2015

Topology Proceedings

Web:	http://topology.auburn.edu/tp/
Mail:	Topology Proceedings Department of Mathematics & Statistics Auburn University, Alabama 36849, USA
E-mail:	topolog@auburn.edu
ISSN:	0146-4124

http://topology.auburn.edu/tp/
http://topology.nipissingu.ca/tp/

TOPOLOGY PROCEEDINGS
Volume 47 (2016)
Pages 89-100

E-Published on May 2, 2015

A FIXED POINT THEOREM FOR CONTRACTING MAPS OF SYMMETRIC CONTINUITY SPACES

NATHANAEL LEEDOM ACKERMAN

Abstract. We prove a generalization of the Banach fixed point theorem for symmetric separated $\mathbb{V}$-continuity spaces. We also give examples to show that in general we cannot weaken our assumptions.

1. Introduction

One of the most important fixed point theorems to arise from the study of metric spaces is the Banach fixed point theorem. This theorem can be stated as follows:

Theorem 1.1 (Banach Fixed Point Theorem). *If (M, d_M) is a Cauchy complete metric space, $m < n \in \mathbb{N}$ and $f : M \to M$ is a function such that*

$$(\forall x, y \in M)\ n \cdot d_M(f(x), f(y)) \leq m \cdot d_M(x, y)$$

then f has a unique fixed point.

It is natural to ask, "On what spaces, other than Cauchy complete metric spaces, and for which classes of maps, can such a fixed point theorem be proved?" In [3] it was shown that for every topological space $(T, \mathcal{O}(T))$ there is a quantale $\mathbb{V}$ and $\mathbb{V}$-continuity space (T, d_T) such that $(T, \mathcal{O}(T))$ is homeomorphic to the topological space of open balls of (T, d_T). This allows us to refine the above question to "For what quantale's $\mathbb{V}$, what $\mathbb{V}$-continuity spaces (M, d_M) and what class of non-expanding maps does a version of the Banach fixed point theorem hold?"

2010 *Mathematics Subject Classification.* 54H25, 47H09, 54E50, 54E50, 18D20.
Key words and phrases. Banach Fixed Point Theorem, Generalized Metric Spaces, Continuity Spaces, Contracting Maps.