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by

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DECOMPOSITIONS OF FUNCTION SPACES

DAVID GUERRERO SÁNCHEZ

Abstract. In this article we generalize a known result of Velichko by proving that a space $C_p(X)$ is the union of less than $\mathfrak{d}$ of its countably compact subspaces if and only if X is finite. We present an example of a space X which is not a P-space and $C_p(X,[0,1])$ admits a closure-preserving cover by countably compact subspaces. It is also proved that $C_p(X,[0,1])$ is contained in the closure of a second countable space $M \subset C_p(X)$ and for some $f \in C_p(X,[0,1])$ the space $M \cup \{f\}$ has a countable local base at f, then X is countable.

1. Introduction

In this note we continue with the work started in [8] and [10] by studying different kind of decompositions of function spaces. In section 3 we look at decompositions that yield covers of function spaces by compact-like subspaces. In this case, we will extend a result of Velichko by showing that $C_p(X)$ is the union of less than $\mathfrak{d}$ of its compact subspaces if and only if X is finite. We will also provide an example of a space X such that $C_p(X,\mathbb{I})$ is not countably compact, but has a closure-preserving cover by countably compact spaces.

Section 4 is devoted to the study of certain topological games in function spaces. Let $\mathcal{C}(\mathcal{P})$ be the class of topological spaces with a certain topological property $\mathcal{P}$. We will see that for many topological properties $\mathcal{P}$ there is a very similar behavior between spaces $C_p(X)$ for which Player I has a winning strategy in the game $\mathcal{G}(\mathcal{C}(\mathcal{P}), C_p(X))$ and those with a closure-preserving cover by subspaces in the class $\mathcal{C}(\mathcal{P})$. However, substantial differences will be observed for the Lindelöf and Lindelöf Σ properties.

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