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TOPOLOGY PROCEEDINGS

Volume 47, 2016

Pages 163–176

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Electronically published on August 9, 2015

Topology Proceedings

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E-mail:	topolog@auburn.edu
ISSN:	0146-4124

http://topology.auburn.edu/tp/
http://topology.nipissingu.ca/tp/

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Volume 47 (2016)
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STRUCTURE SPACES OF INTERMEDIATE RINGS OF ORDERED FIELD VALUED CONTINUOUS FUNCTIONS

SUDIP KUMAR ACHARYYA AND PRITAM ROOJ

Abstract. Let F be a totally ordered field equipped with its order topology and X, a Hausdorff Completely F-regular topological space(CFR space in short) in the sense that, points and closed sets in X could be separated by F-valued continuous functions on X. Suppose $C(X,F)$ is the ring of all F-valued continuous functions on X and $B(X,F) = \{f \in C(X,F) : |f| < \lambda \text{ for some } \lambda > 0 \text{ in } F\}$. We call any ring $A(X,F)$ lying between $B(X,F)$ and $C(X,F)$ an intermediate ring. Given an intermediate ring $A(X,F)$ it is shown that, there is a one-to-one correspondence between the set $\mathcal{M}_F(A)$ of all maximal ideals in this ring and the set $\beta_F X$ of all z_F-ultrafilters on X. If $\mathcal{M}_F(A)$ is endowed with the Hull-Kernel topology and $\beta_F X$ with the Stone topology, then these two spaces become homeomorphic. This extends a result of Byun and Watson [3] which says on choosing $F = \mathbb{R}$ that, the structure space of any ring lying between $C^*(X)$ and $C(X)$ is βX, the Stone-Čech compactification of X. The Hausdorff compactification $\beta_F X$ of X thus obtained enjoys a kind of extension property similar to that of βX described as follows: any continuous map from X to a compact Hausdorff CFR space Y extends to a continuous map from $\beta_F X$ to Y. Using this extension property, we have shown that the ring $C_K(X,F)$ of all functions in $C(X,F)$ with compact support becomes identical to the set $\bigcap_{p\in\beta_F X - X} O^p_F$, where for $p \in \beta_F X$, $O^p_F = \{f \in C(X,F)$: the closure in $\beta_F X$ of the zero-set of f in X is a neighborhood of p in the space $\beta_F X$ $\}$. A special case of this result with $F = \mathbb{R}$ yields the standard formula $C_K(X) = \bigcap_{p\in\beta X - X} O^p$ in the classical situation. This exemplifies a further similarity between $\beta_F X$ and βX.

2010 *Mathematics Subject Classification.* Primary 54C40; Secondary 46E25.

Key words and phrases. Totally ordered field, zero-dimensional space, compact support, intermediate ring, hull-kernel topology, stone topology, extension property.

The first author is the corresponding author.

The second author thanks the UGC, New Delhi-110002, India, for financial support.