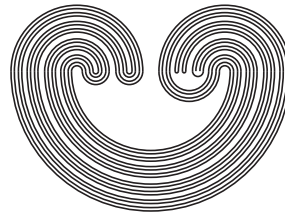


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## STRUCTURE SPACES OF INTERMEDIATE RINGS OF ORDERED FIELD VALUED CONTINUOUS FUNCTIONS

by

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## STRUCTURE SPACES OF INTERMEDIATE RINGS OF ORDERED FIELD VALUED CONTINUOUS FUNCTIONS

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**ABSTRACT.** Let  $F$  be a totally ordered field equipped with its order topology and  $X$ , a Hausdorff Completely  $F$ -regular topological space (CFR space in short) in the sense that, points and closed sets in  $X$  could be separated by  $F$ -valued continuous functions on  $X$ . Suppose  $C(X, F)$  is the ring of all  $F$ -valued continuous functions on  $X$  and  $B(X, F) = \{f \in C(X, F) : |f| < \lambda \text{ for some } \lambda > 0 \text{ in } F\}$ . We call any ring  $A(X, F)$  lying between  $B(X, F)$  and  $C(X, F)$  an intermediate ring. Given an intermediate ring  $A(X, F)$  it is shown that, there is a one-to-one correspondence between the set  $\mathcal{M}_F(A)$  of all maximal ideals in this ring and the set  $\beta_F X$  of all  $z_F$ -ultrafilters on  $X$ . If  $\mathcal{M}_F(A)$  is endowed with the Hull-Kernel topology and  $\beta_F X$  with the Stone topology, then these two spaces become homeomorphic. This extends a result of Byun and Watson [3] which says on choosing  $F = \mathbb{R}$  that, the structure space of any ring lying between  $C^*(X)$  and  $C(X)$  is  $\beta X$ , the Stone-Ćech compactification of  $X$ . The Hausdorff compactification  $\beta_F X$  of  $X$  thus obtained enjoys a kind of extension property similar to that of  $\beta X$  described as follows: any continuous map from  $X$  to a compact Hausdorff CFR space  $Y$  extends to a continuous map from  $\beta_F X$  to  $Y$ . Using this extension property, we have shown that the ring  $C_K(X, F)$  of all functions in  $C(X, F)$  with compact support becomes identical to the set  $\bigcap_{p \in \beta_F X - X} O_F^p$ , where for  $p \in \beta_F X$ ,  $O_F^p = \{f \in C(X, F) : \text{the closure in } \beta_F X \text{ of the zero-set of } f \text{ in } X \text{ is a neighborhood of } p \text{ in the space } \beta_F X\}$ . A special case of this result with  $F = \mathbb{R}$  yields the standard formula  $C_K(X) = \bigcap_{p \in \beta X - X} O^p$  in the classical situation. This exemplifies a further similarity between  $\beta_F X$  and  $\beta X$ .

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