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TOPOLOGY PROCEEDINGS

Volume 47, 2016

Pages 191–205

http://topology.nipissingu.ca/tp/

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by

Paloma Hernández and Héctor Méndez

Electronically published on October 11, 2015



**Topology Proceedings**

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| **E-mail:** | topolog@auburn.edu |
| **ISSN:** | 0146-4124 |

http://topology.auburn.edu/tp/
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Volume 47 (2016)
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E-Published on October 11, 2015

# ENTROPY OF INDUCED DENDRITE HOMEOMORPHISMS

PALOMA HERNÁNDEZ AND HÉCTOR MÉNDEZ

Abstract. Let $f : D \to D$ be a dendrite homeomorphism. Let $2^D$ denote the hyperspace of all nonempty compact subsets of $D$ endowed with the Hausdorff metric. Let $2^f : 2^D \to 2^D$ be the induced homeomorphism. We show in this note that the topological entropy of $2^f$ has only two possible values: 0 or $\infty$. This claim generalizes a result due to M. Lampart and P. Raith.

## 1. Introduction and some definitions

A *continuum* is a nonempty compact and connected metric space.

Let $X = (X, d)$ be a continuum. Let $2^X$ be the collection of all nonempty compact subsets of $X$ endowed with the Hausdorff metric $H_d$ induced by metric $d$. If $Y$ is a continuum and $Y \subset X$ then $Y$ is a *subcontinuum* of $X$.

It is said that $X$ is

- an *arc* provided that it is homeomorphic to the unit interval $[0, 1]$,
- a *simple closed curve* provided that it is homeomorphic to the circle $S^1 = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 = 1\}$,
- a *dendrite* provided that it is a locally connected and contains no simple closed curves.

Let $\mathbb{N}$ denote the set of all positive integers. A *mapping* is a continuous function. Let $f : X \to X$ be a mapping.

Let $2^f : 2^X \to 2^X$ be the mapping induced in $2^X$ by $f$. For each $n \in \mathbb{N}$ and for each $A \in 2^X$, $\left(2^f\right)^n (A) = f^n(A)$.

2010 *Mathematics Subject Classification.* Primary 37B40, 37B45; Secondary 54B20.

*Key words and phrases.* Topological entropy, omega limit set, dendrite homeomorphism, induced hyperspace homeomorphism.