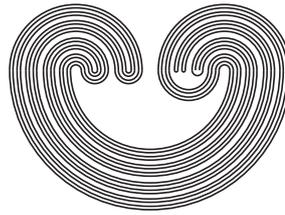


<http://topology.auburn.edu/tp/>

---

# TOPOLOGY PROCEEDINGS



Volume 47, 2016

Pages 221–259

---

<http://topology.nipissingu.ca/tp/>

ON THE SUBMETRIZABILITY NUMBER AND  
 $i$ -WEIGHT OF QUASI-UNIFORM SPACES AND  
PARATOPOLOGICAL GROUPS

by

TARAS BANAKH AND ALEX RAVSKY

Electronically published on October 17, 2015

This file contains only the first page of the paper. The full version of the paper is available to Topology Proceedings subscribers. See <http://topology.auburn.edu/tp/subscriptioninfo.html> for information.

---

**Topology Proceedings**

**Web:** <http://topology.auburn.edu/tp/>

**Mail:** Topology Proceedings  
Department of Mathematics & Statistics  
Auburn University, Alabama 36849, USA

**E-mail:** [topolog@auburn.edu](mailto:topolog@auburn.edu)

**ISSN:** 0146-4124

COPYRIGHT © by Topology Proceedings. All rights reserved.

**ON THE SUBMETRIZABILITY NUMBER AND  
 $i$ -WEIGHT OF QUASI-UNIFORM SPACES AND  
PARATOPOLOGICAL GROUPS**

TARAS BANAKH AND ALEX RAVSKY

**ABSTRACT.** We derive many upper bounds on the submetrizability number and  $i$ -weight of paratopological groups and topological monoids with open shifts. In particular, we prove that each first countable Hausdorff paratopological group is submetrizable thus answering a problem of Arhangel'skii posed in 2002. Also we construct an example of a zero-dimensional (and hence regular) Hausdorff paratopological abelian group  $G$  with countable pseudocharacter which is not submetrizable. In fact, all results on the  $i$ -weight and submetrizability are derived from more general results concerning normally quasi-uniformizable and bi-quasi-uniformizable spaces.

**INTRODUCTION**

This paper was motivated by the following problem of Arhangel'skii [1, 3.11] (also repeated by Tkachenko in his survey [26, 2.1]): *Does every first countable Hausdorff paratopological group admit a weaker metrizable topology?* A surprisingly simple answer to this problem was given by the authors in [4]. We just observed that each Hausdorff paratopological group  $G$  carries a natural uniformity generated by the base consisting of entourages  $\{(x, y) \in G \times G : y \in UxU^{-1} \cap U^{-1}xU\}$  where  $U$  runs over open neighborhoods of the unit  $e$  in  $G$ . In [4] this uniformity was called the *quasi-Roelcke uniformity* on  $G$  and denoted by  $\mathcal{Q}$ . If  $G$  is first-countable, then the quasi-Roelcke uniformity  $\mathcal{Q}$  is metrizable, which implies that the space  $G$  is submetrizable. Moreover, if the quasi-Roelcke uniformity  $\mathcal{Q}$  is

---

2010 *Mathematics Subject Classification.* 54D10, 54D15, 54E15, 22A30.

*Key words and phrases.* Submetrizable space,  $i$ -weight, pre-uniformity, quasi-uniformity, paratopological group, topological monoid.

The first author has been partially financed by NCN grant DEC-2012/07/D/ST1/02087.

©2015 Topology Proceedings.

This file contains only the first page of the paper. The full version of the paper is available to Topology Proceedings subscribers. See <http://topology.auburn.edu/tp/subscriptioninfo.html> for information.