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UNIFORM BOX PRODUCTS II

by

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ABSTRACT. The uniform box topology is defined on the product set of infinitely many copies of a completely regular space. It is finer than the Tychonoff topology but coarser than the box topology. Here we study connectedness and separation properties.

1. INTRODUCTION

The uniform box product, introduced by Scott W. Williams in 2001, is a generalization of the sup metric topology on the product of metric spaces to powers of uniform spaces. Its topology is finer than the Tychonoff but coarser than the box topology. Many questions open for box products [9] are open for uniform box products as well.

On a compact space, there is a unique uniformity which generates the topology. However, when a uniform space is not compact, there may be many uniformities which generate the topology. Even if two uniformities generate the same topology on a space, their respective uniform box products may differ. For example, in section 4 we show that the uniform box product of copies of the real line is not connected when we use its canonical uniformity, yet it is connected when we use the uniformity of a homeomorph. The situation is somewhat less complex if we restrict our attention to uniformities induced by a compactification (section 3) and which are totally bounded. One consequence of our approach is that, by restricting to totally bounded uniformities, the uniform box product of a connected uniform space is connected.

²⁰¹⁰ Mathematics Subject Classification. Primary 54B10, 54E15; Secondary 54D15, 54D05, 54D45, 54G12.

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