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by

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ON TOPOLOGICAL COMPLEXITY OF EILENBERG–MACLANE SPACES

YULI RUDYAK

Abstract. We note that, for any natural k and every natural l between k and $2k$, there exists a group π with $\operatorname{cat} K(\pi, 1) = k$ and $\mathrm{TC}(K(\pi, 1)) = l$. Because of this, we can set up a problem for searching for a purely group-theoretical description of $\mathrm{TC}(K(\pi, 1))$ as an invariant of π.

Below $\operatorname{cat} X$ denotes the Lusternik–Schnirelmann category (normalized, i.e., $\operatorname{cat} S^n = 1$, see [2]). Furthermore, we denote by $\mathrm{TC}(X)$ the topological complexity of X defined by Michael Farber [5], but we use the normalized version in [7], [8].

Because of results of Alexander Dranishnikov [3, Lemma 2.7 and Theorem 3.6], we get the following inequalities:

$$\operatorname{cat}(G \times H) \leq \mathrm{TC}(G \vee H) \leq \operatorname{cat} G + \operatorname{cat} H. \tag{1}$$

Farber asked about calculation of $\mathrm{TC}(K(\pi, 1))$'s. It is known that $\operatorname{cat} X \leq \mathrm{TC}(X) \leq \operatorname{cat}(X \times X)$ for all X [5]. The following observation tells us that, in the class of $K(\pi, 1)$-spaces, the above mentioned inequality gets no new bounds.

Theorem 1. *For every natural k and every natural l with $k \leq l \leq 2k$, there exists a discrete group π such that π with $\operatorname{cat} K(\pi, 1) = k$ and $\mathrm{TC}(K(\pi, 1)) = l$. In fact, we can put $\pi = \mathbb{Z}^k * \mathbb{Z}^{l-k}$.*

Proof. Let T^m be the m-torus. Then $\operatorname{cat} T^m = m$. Put $r = l - k$ and consider the free product $\pi := \mathbb{Z}^k * \mathbb{Z}^r$. Then $K(\pi, 1) = T^k \vee T^r$, because

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