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ENDPOINTS OF INVERSE LIMITS WITH SET-VALUED FUNCTIONS

JAMES P. KELLY

ABSTRACT. Suppose that $\{\mathbf{X}, \mathbf{F}\}$ is an inverse sequence where, for each $i \in \mathbb{N}$, $F_i : X_{i+1} \to 2^{X_i}$ is upper semi-continuous, and suppose that $\mathbf{p}$ is a point of the inverse limit of this inverse sequence. We show that $\mathbf{p}$ is an endpoint of $\varprojlim \mathbf{F}$ provided that for infinitely many $n \in \mathbb{N}$, $(p_1, \ldots, p_n)$ is an endpoint of $\Gamma_n = \{\mathbf{x} \in \prod_{i=1}^n X_i : x_i \in F_i(x_{i+1}) \text{ for } 1 \leq i < n\}$.

Additionally, in the special case that each bonding function has its inverse equal to the union of mappings, we show that $\mathbf{p}$ is an endpoint of $\varprojlim \mathbf{F}$ if and only if $(p_1, \ldots, p_n)$ is an endpoint of Γ_n for all $n \in \mathbb{N}$. We show some examples of how this characterization of endpoints of an inverse limit may be used to show that two inverse limits are not homeomorphic.

We also demonstrate how these results may be applied to inverse limits with continuous, single-valued bonding functions.

INTRODUCTION

We begin with some definitions for the terms used in this paper.

A *compactum* is a non-empty, compact metric space. A *continuum* is a connected compactum. A continuum which is a subset of a compactum X is called a *subcontinuum* of X.

A point p in a compactum X is called an *endpoint* of X if, for any two subcontinua H and K of X which both contain p, either $H \subseteq K$ or $K \subseteq H$.

Given a compactum X, we define 2^X to be the space consisting of all non-empty, compact subsets of X. Given a function $F : X \to 2^Y$, we

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