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A CLASS OF EVEN ELLIPTIC FUNCTIONS WITH NO HERMAN RINGS

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Abstract. In this note we study the dynamical and topological properties of Julia and Fatou sets of certain even elliptic functions. By computing their conformal class, we obtain sufficient conditions to show these functions do not exhibit Herman rings, extending known results for the Weierstrass $\wp$-function (Jane Hawkins and Lorelei Koss, *Parametrized dynamics of the Weierstrass elliptic function*, Conform. Geom. Dyn. **8** (2004)), and for $1/\wp$ over triangular lattices (Koss, *A fundamental dichotomy for Julia sets of a family of elliptic functions*, Proc. Amer. Math. Soc. **137** (2009), no. 11). As an application, we show Julia sets of $1/\wp$ over square lattices are either connected or totally disconnected.

1. Introduction

Consider an analytic map $f : U \to U$ defined over an annular domain U of the complex plane. U is called a *Herman ring* if the iterates of $f|U$ are analytically conjugate to an irrational rotation acting on a round, non-degenerated annulus. In general, it is a difficult problem to determine their existence since, in contrast to other types of Fatou domains, Herman rings are not associated to periodic orbits.

In this note we give sufficient conditions on a class of even elliptic functions that prevents the presence of cycles of Herman rings. Elliptic functions can be described succinctly as double periodic transcendental meromorphic functions whose set of poles form an infinite and discrete set

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