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HALF-TURN LINKED PAIRS OF ISOMETRIES OF HYPERBOLIC 4-SPACE

ANDREW E. SILVERIO

Abstract. In this paper we develop a complete theory of factorization for isometries of hyperbolic 4-space. Of special interest is the case where a pair of isometries is linked, that is, when a pair of isometries can be expressed each as compositions of two involutions, one of which is common to both isometries.

Here we develop a new theory of hyperbolic pencils and twisting planes involving a new geometric construction, their half-turn banks. This enables us to give complete results about each of the pair-types of isometries and their simultaneous factorization by half-turns. That is, we provide geometric conditions for each such pair to be linked by half-turns. The main result gives a necessary and sufficient condition for any given pair of isometries to be linked. We also provide a procedure for constructing a half-turn linked pair of isometries of $\mathbb{H}^4$ that do not restrict to lower dimensions, yielding an example that gives a negative answer to a question raised by Ara Basmajian and Karan Puri.

1. Introduction

A pair of isometries A and B of the hyperbolic space $\mathbb{H}^n$ is said to be linked if there are involutions α, β, and γ such that $A = \alpha\beta$ and $B = \beta\gamma$. If, furthermore, α, β, and γ have a $(n-2)$-dimensional fixed-point set, then the pair is said to be linked by half-turns. In dimensions 2 and 3, every pair of isometries is linked. Factoring by half-turns is used to determine the discreteness of the group $\langle A, B\rangle$ using the Gilman-Maskit algorithm in dimension 2, the non-separating-disjoint-circles condition, and the compact-core-geodesic-intersection condition in dimension 3 [4],

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