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by

SHARAN GOPAL

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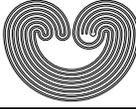
Department of Mathematics & Statistics

Auburn University, Alabama 36849, USA

E-mail: topolog@auburn.edu

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TRAJECTORIES OF CHAOTIC INTERVAL MAPS

SHARAN GOPAL

ABSTRACT. This paper proves the existence of an abundant number of sequences in $[0, 1]$ that can occur as trajectories for chaotic interval maps. It is proved that given an allowed sequence in $[0, 1]$ of certain kind, there always exists a chaotic interval map with this sequence as a trajectory.

1. INTRODUCTION

By a *dynamical system* (X, f) , we mean a topological space X and a continuous self map f on it. The *trajectory* of a point $x \in X$ is the sequence $(x, f(x), f^2(x), \dots)$, where $f^n = f \circ f \circ \dots \circ f$ (n times) and the set $\{f^n(x) : n \in \mathbb{N}_0\}$ is called the *orbit* of x (\mathbb{N}_0 is the set of non-negative integers and $f^0(x) = x$). If $f^n(x) = x$ for some $n \in \mathbb{N}$, then x is called a *periodic point*. An *interval map* is a dynamical system, where the underlying topological space is $[0, 1]$, i.e., a system of the form $([0, 1], f)$.

Definition 1.1 (See [3]). Let (X, f) be a dynamical system, where X is a metric space with metric d . (X, f) is said to be *Devaney chaotic* if

- (1) f has sensitive dependence on initial conditions (i.e., there is an $r > 0$ such that for each point $x \in X$ and for each $\epsilon > 0$ there is a point $y \in X$ with $d(x, y) < \epsilon$ and a $k \geq 0$ such that $d(f^k(x), f^k(y)) \geq r$),

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