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TREE-LIKE INVERSE LIMITS ON [0,1] WITH INTERVAL-VALUED FUNCTIONS

M. M. MARSH

Abstract. We investigate dimension one, tree-likeness, and dimension greater than one in inverse limits on $[0, 1]$ with interval-valued bonding functions. Our investigation leads to some generalizations of results of W. T. Ingram and to necessary conditions and sufficient conditions for tree-likeness in this setting.

In recent papers [9], [10], [11], W. T. Ingram has determined a number of sufficient conditions for inverse limits with set-valued bonding functions to be 1-dimensional, and in many cases, to be tree-like. We generalize some of Ingram's results and establish necessary conditions and sufficient conditions for tree-likeness of inverse limits on $[0, 1]$ with interval-valued bonding functions. We also establish necessary and sufficient conditions for the emergence of dimension greater than one in the sets $G'(f_1, \ldots, f_n)$. Our conditions involve the notion of flat spots for the bonding functions and whether the flat spots compose to nondegenerate values of earlier bonding functions in the inverse sequence. Ingram introduced these concepts in the papers referenced above and he states at the end of section 3 in [11] that it would be interesting to know if the only way that the graph of a composition of a sequence of interval-valued functions can have dimension greater than one is for some flat spot for a term of the inverse sequence to iterate to a point where an earlier term of the sequence has a nondegenerate value. Example 14 shows that this does not have to be the case. That is, the graph of a composition can have dimension two even though no flat spot composes to a nondegenerate value. However,

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