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by

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A FINITE PRESENTATION FOR THE AUTOMORPHISM GROUP OF THE FIRST HOMOLOGY OF A NON-ORIENTABLE SURFACE OVER $\mathbb{Z}_2$ PRESERVING THE MOD 2 INTERSECTION FORM

RYOMA KOBAYASHI AND GENKI OMORI

Abstract. Let $\mathrm{Aut}(H_1(N_g;\mathbb{Z}_2),\cdot)$ be the group of automorphisms on the first homology group with $\mathbb{Z}_2$ coefficients of a closed non-orientable surface N_g preserving the mod 2 intersection form. In this paper, we obtain a finite presentation for $\mathrm{Aut}(H_1(N_g;\mathbb{Z}_2),\cdot)$. As an application we calculate the second homology group of $\mathrm{Aut}(H_1(N_g;\mathbb{Z}_2),\cdot)$.

1. Introduction

For $g \geq 1$ and $n \geq 0$, let $N_{g,n}$ be a compact connected non-orientable surface of genus g with n boundary components (we denote $N_{g,0}$ by N_g) and a bilinear form $\cdot : H_1(N_g;\mathbb{Z}_2) \times H_1(N_g;\mathbb{Z}_2) \to \mathbb{Z}_2$ the mod 2 intersection form on the first homology group $H_1(N_g;\mathbb{Z}_2)$ of N_g with $\mathbb{Z}_2$ coefficients. We represent N_g by a sphere with g crosscaps as in Figure 1; i.e., we regard N_g as a sphere with g boundary components and a Möbius band attached to each boundary component. We define $\mathrm{Aut}(H_1(N_g;\mathbb{Z}_2),\cdot)$ by the subgroup of the automorphism group $\mathrm{Aut}\,H_1(N_g;\mathbb{Z}_2)$ of $H_1(N_g;\mathbb{Z}_2)$ preserving the mod 2 intersection form $\cdot$. Note that $\mathrm{Aut}(H_1(N_g;\mathbb{Z}_2),\cdot)$ is isomorphic to $O(g,\mathbb{Z}_2) = \{A \in GL(g,\mathbb{Z}_2) \mid {}^tAA = E\}$ by taking the basis $\{x_1, x_2, \ldots, x_g\}$ for $H_1(N_g;\mathbb{Z}_2)$, where x_i is a homology class of a one-sided simple closed curve μ_i in Figure 1 and E is an identity matrix

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