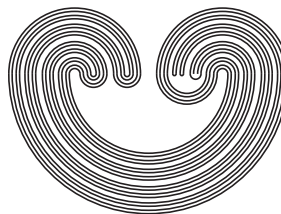


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A FINITE PRESENTATION FOR THE
AUTOMORPHISM GROUP OF THE FIRST
HOMOLOGY OF A NON-ORIENTABLE
SURFACE OVER \mathbb{Z}_2 PRESERVING THE MOD 2
INTERSECTION FORM

by

RYOMA KOBAYASHI AND GENKI OMORI

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**A FINITE PRESENTATION FOR THE AUTOMORPHISM
GROUP OF THE FIRST HOMOLOGY OF A
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THE MOD 2 INTERSECTION FORM**

RYOMA KOBAYASHI AND GENKI OMORI

ABSTRACT. Let $\text{Aut}(H_1(N_g; \mathbb{Z}_2), \cdot)$ be the group of automorphisms on the first homology group with \mathbb{Z}_2 coefficients of a closed non-orientable surface N_g preserving the mod 2 intersection form. In this paper, we obtain a finite presentation for $\text{Aut}(H_1(N_g; \mathbb{Z}_2), \cdot)$. As an application we calculate the second homology group of $\text{Aut}(H_1(N_g; \mathbb{Z}_2), \cdot)$.

1. INTRODUCTION

For $g \geq 1$ and $n \geq 0$, let $N_{g,n}$ be a compact connected non-orientable surface of genus g with n boundary components (we denote $N_{g,0}$ by N_g) and a bilinear form $\cdot \cdot : H_1(N_g; \mathbb{Z}_2) \times H_1(N_g; \mathbb{Z}_2) \rightarrow \mathbb{Z}_2$ the mod 2 intersection form on the first homology group $H_1(N_g; \mathbb{Z}_2)$ of N_g with \mathbb{Z}_2 coefficients. We represent N_g by a sphere with g crosscaps as in Figure 1; i.e., we regard N_g as a sphere with g boundary components and a Möbius band attached to each boundary component. We define $\text{Aut}(H_1(N_g; \mathbb{Z}_2), \cdot)$ by the subgroup of the automorphism group $\text{Aut } H_1(N_g; \mathbb{Z}_2)$ of $H_1(N_g; \mathbb{Z}_2)$ preserving the mod 2 intersection form $\cdot \cdot$. Note that $\text{Aut}(H_1(N_g; \mathbb{Z}_2), \cdot)$ is isomorphic to $O(g, \mathbb{Z}_2) = \{A \in GL(g, \mathbb{Z}_2) \mid {}^tAA = E\}$ by taking the basis $\{x_1, x_2, \dots, x_g\}$ for $H_1(N_g; \mathbb{Z}_2)$, where x_i is a homology class of a one-sided simple closed curve μ_i in Figure 1 and E is an identity matrix

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