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by

Samer Assaf and Koushik Pal

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Web:	http://topology.auburn.edu/tp/
Mail:	Topology Proceedings Department of Mathematics & Statistics Auburn University, Alabama 36849, USA
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PARTIAL METRIC SPACES WITH NEGATIVE DISTANCES AND FIXED POINT THEOREMS

SAMER ASSAF AND KOUSHIK PAL

Abstract. In this paper we consider partial metric spaces in the sense of O'Neill. We introduce the notions of strong partial metric spaces and Cauchy functions. We prove a fixed point theorem for such spaces and functions that improves Matthews' contraction mapping theorem in two ways. First, the existence of fixed points now holds for a wider class of functions and spaces. Second, our theorem also allows for fixed points with nonzero self-distances. We also prove fixed point theorems for orbitally r-contractive and orbitally ϕ_r-contractive maps. We then apply our results to give alternative proofs of some of the other known fixed point theorems in the context of partial metric spaces.

1. Introduction

The notion of *distance* is fundamental in mathematics and variations on distance have been much studied (see [5]). One such variation, the *partial metric*, was introduced by Matthews (see [10, 11]). It differs from a metric in that points are allowed to have nonzero "self-distances" (i.e., $d(x, x) \geq 0$) and the triangle inequality is modified to account for positive self-distance. The notion of a partial metric has been both fruitful and well-studied (see http://www.dcs.warwick.ac.uk/pmetric/index.html).

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Key words and phrases. Partial metric, strong partial metric, negative distance, Cauchy map, Cauchy mapping theorem, fixed point theorems.

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