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AMALGAMATION-TYPE PROPERTIES OF ARCS AND PSEUDO-ARCS

PAUL BANKSTON

Abstract. A continuum X is **base** if it satisfies the following "dual amalgamation" condition: whenever $f : Y \to X$ and $g : Z \to X$ are continuous maps from continua onto X, there is a continuum W and continuous surjections $\varphi : W \to Y$, $\gamma : W \to Z$ such that $f \circ \varphi = g \circ \gamma$. A metrizable continuum is **base metrizable** if it satisfies the condition above, relativized to the subclass of metrizable continua. It is easy to show that simple closed curves are neither base nor base metrizable; however metrizable continua of span zero are known to be base metrizable. Furthermore, co-existentially closed continua are known to be base. The arc and the pseudo-arc are span zero; but, of the two, only the pseudo-arc is co-existentially closed. Hence the pseudo-arc is base metrizable for being span zero and base for being co-existentially closed. Here we show that: (i) there is a base metrizable continuum which is not span zero; and (ii) any metrizable continuum is base if and only if it is base metrizable.

1. Introduction

In algebra and model theory a structure A in a certain class $\mathfrak{C}$ is referred to as an **amalgamation base** for $\mathfrak{C}$ if whenever A sits as a substructure of two members of $\mathfrak{C}$, there is a third member of $\mathfrak{C}$ which contains all of them. There are many natural variations on this theme; the one we consider here has continua instead of relational structures and quotients

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