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METRIC CONSTRUCTIONS OF TOPOLOGICAL INVARIANTS

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Abstract. We present a general mechanism for obtaining topological invariants from metric constructs. In more detail, we describe a process which, under very mild conditions, produces topological invariants out of a construction on a metric space together with a choice of scale (a non-negative value at each point of the space). Through Flagg's metric formalism of topology the results are valid for all topological spaces, not just the metrizable ones. We phrase the result in much greater generality than required for the topological applications, using the language of fibrations. We show that ordinary topological connectedness arises metrically, and we obtain metrically defined theories of homology and of homotopy.

1. Introduction

The classical definition of metric space makes explicit use of $\mathbb{R}$ as the codomain of the metric function. However, we note that for the immediate formulation of the axioms of a metric space, all that is required is the existence of $0 \in \mathbb{R}$, the ordering on $\mathbb{R}$ (but not its linearity), and the operation of addition. Surely some further algebraic properties play a role, but it is not of any significant importance from a metric point-of-view that the metric function takes values in an ordered field - the field axioms are unnecessarily strong and the ordering unnecessarily linear. Further, only slightly beneath the surface of the theory of metric spaces lies the

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