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BOUNDEDNESS OF THE RELATIVES OF UNIFORMLY CONTINUOUS FUNCTIONS

MANISHA AGGARWAL AND S. KUNDU

Abstract. A function f from a metric space (X, d) to another metric space (Y, ρ) is said to be Cauchy-continuous if $(f(x_n))$ is Cauchy in (Y, ρ) for every Cauchy sequence (x_n) in (X, d). Recently in [5], Beer and Garrido have characterized those metric spaces (X, d) on which each Cauchy-continuous function defined on X is bounded. Since in the literature, we have various other kinds of sequences that are weaker than Cauchy sequences, in this paper we have discussed a few properties of functions preserving different kinds of sequences and characterized those metric spaces on which each such function is bounded. It suffices in each case to consider real-valued functions. We observe that a uniformly continuous function preserves all those sequences, so those aforesaid functions are actually the relatives of uniformly continuous functions.

1. Introduction

The concepts of compactness and completeness play a vital role in the theory of metric spaces. Surely for discussing completeness of a metric space, one has to consider its corresponding Cauchy sequences. We recall that a sequence (x_n) in (X, d) is said to be Cauchy if for every $\epsilon > 0$, there exists $n_o \in \mathbb{N}$ such that for each $n, j \geq n_o$, we have $d(x_n, x_j) < \epsilon$. Some classes of metric spaces satisfying properties stronger than completeness

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