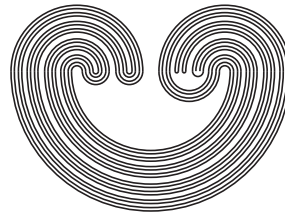


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## BOUNDEDNESS OF THE RELATIVES OF UNIFORMLY CONTINUOUS FUNCTIONS

by

MANISHA AGGARWAL AND S. KUNDU

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## BOUNDEDNESS OF THE RELATIVES OF UNIFORMLY CONTINUOUS FUNCTIONS

MANISHA AGGARWAL AND S. KUNDU

**ABSTRACT.** A function  $f$  from a metric space  $(X, d)$  to another metric space  $(Y, \rho)$  is said to be Cauchy-continuous if  $(f(x_n))$  is Cauchy in  $(Y, \rho)$  for every Cauchy sequence  $(x_n)$  in  $(X, d)$ . Recently in [5], Beer and Garrido have characterized those metric spaces  $(X, d)$  on which each Cauchy-continuous function defined on  $X$  is bounded. Since in the literature, we have various other kinds of sequences that are weaker than Cauchy sequences, in this paper we have discussed a few properties of functions preserving different kinds of sequences and characterized those metric spaces on which each such function is bounded. It suffices in each case to consider real-valued functions. We observe that a uniformly continuous function preserves all those sequences, so those aforesaid functions are actually the relatives of uniformly continuous functions.

### 1. INTRODUCTION

The concepts of compactness and completeness play a vital role in the theory of metric spaces. Surely for discussing completeness of a metric space, one has to consider its corresponding Cauchy sequences. We recall that a sequence  $(x_n)$  in  $(X, d)$  is said to be Cauchy if for every  $\epsilon > 0$ , there exists  $n_o \in \mathbb{N}$  such that for each  $n, j \geq n_o$ , we have  $d(x_n, x_j) < \epsilon$ . Some classes of metric spaces satisfying properties stronger than completeness

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