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by

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EXTENDING T_1 TOPOLOGIES TO HAUSDORFF WITH THE SAME SETS OF LIMIT POINTS

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Abstract. Within the framework of **ZF** set theory we show that the statements: "Every infinite T_1 topological space (X, Q) with a finite set of limit points can be extended to a T_2 space with the same set of limit points" and "there exist no free ultrafilters" are equivalent.

1. Notation and terminology

Given a set X, a non-empty collection $\mathcal{F} \subseteq \mathcal{P}(X)\backslash\{\varnothing\}$ is called a *filter* iff it is closed under finite intersections and for every $F \in \mathcal{F}$ and $O \subseteq X$ if $F \subseteq O$ then $O \in \mathcal{F}$.

A non-empty collection $\mathcal{H} \subseteq \mathcal{P}(X)\backslash\{\varnothing\}$ is a *filterbase* iff it is closed under finite intersections.

A filterbase $\mathcal{F}$ of X is called *free* if $\bigcap \mathcal{F} = \varnothing$. A maximal with respect to inclusion filter of X is called *ultrafilter*. $cof(X)$ will denote the filter of all cofinite subsets of X. i.e., $A \in cof(X)$ iff $|X\backslash A| < \aleph_0$.

Let $\mathbf{X} = (X, T)$ be a topological space and $A \subseteq X$. An element $x \in X$ is said to be a *limit point* of A iff for every neighborhood V_x of x, $V_x \cap A\backslash\{x\} \neq \varnothing$. A non-limit point of X is called *isolated*. $Lim_T(X)$ denotes the set of all limit points of $\mathbf{X}$ and $Iso_T(X)$ denotes the set of all isolated points of $\mathbf{X}$. If no confusion is likely to arise we shall omit the subscript T from $Iso_T(X)$ and $Lim_T(X)$. If $A \subseteq X$ then T_A will denote the topology A inherits as a subspace of $\mathbf{X}$.

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