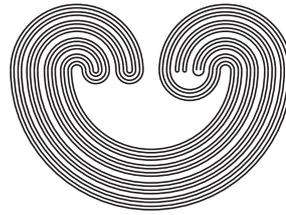


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ASYMMETRY IN MANY-VALUED TOPOLOGY: SPECTRA OF QUANTALES AND SEMIQUANTALES

by

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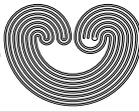
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**ASYMMETRY IN MANY-VALUED TOPOLOGY:
SPECTRA OF QUANTALES AND SEMIQUANTALES**

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AND STEPHEN E. RODABAUGH*

ABSTRACT. The symmetry vis-a-vis asymmetry issue naturally occurs in two settings of traditional topology: the symmetry axiom of a metric space, and the symmetry conditions satisfied by specialization orders associated with certain topological spaces. In each case, the issue of symmetry relates to separation: each metric space is T_2 and symmetry is explicitly invoked for the disjointness of separating neighborhoods; and for T_0 topological spaces, specialization order symmetry is equivalent to T_1 . This paper studies asymmetry for many-valued or L -topological spaces—with various conditions imposed on L —via two “standard” specialization orders (and their duals) associated with such spaces; and special emphasis is placed on spaces which are L -spectra of semiquantales, sometimes with additional restrictions on L or the represented semiquantales; these additional restrictions may involve left- and/or right-residuations as well as with special involutive, isotone, anti-automorphisms. For consistent L : the L -spectrum of a semiquantale with at least two related distinct primes is asymmetric; the left and right topologies of \mathbb{R} and the left and right L -topologies of the “fuzzy” real line $\mathbb{R}(L)$ are all asymmetric; and the L -spectrum of each of these (L -)topologies is asymmetric. On the other hand, for each (complete) DeMorgan algebra L , $\mathbb{R}(L)$ with the canonical topology $\tau(L)$ is symmetric w.r.t. the first specialization order; and the “alternative” fuzzy real line $\mathbb{R}^*(L)$ —the L -spectrum of the standard topology on \mathbb{R} —is symmetric in the same sense if L is a complete Boolean algebra. Under appropriate conditions, the well-known G_χ , M_L , ω_L , ι_L functors produce and/or reflect both asymmetric and symmetric L -topological spaces.

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