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ASYMMETRY IN MANY-VALUED TOPOLOGY: SPECTRA OF QUANTALES AND SEMIQUANTALES

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Abstract. The symmetry vis-a-vis asymmetry issue naturally occurs in two settings of traditional topology: the symmetry axiom of a metric space, and the symmetry conditions satisfied by specialization orders associated with certain topological spaces. In each case, the issue of symmetry relates to separation: each metric space is T_2 and symmetry is explicitly invoked for the disjointness of separating neighborhoods; and for T_0 topological spaces, specialization order symmetry is equivalent to T_1. This paper studies asymmetry for many-valued or L-topological spaces—with various conditions imposed on L—via two "standard" specialization orders (and their duals) associated with such spaces; and special emphasis is placed on spaces which are L-spectra of semiquantales, sometimes with additional restrictions on L or the represented semiquantales; these additional restrictions may involve left- and/or right-residuations as well as with special involutive, isotone, anti-automorphisms. For consistent L: the L-spectrum of a semiquantale with at least two related distinct primes is asymmetric; the left and right topologies of $\mathbb{R}$ and the left and right L-topologies of the "fuzzy" real line $\mathbb{R}(L)$ are all asymmetric; and the L-spectrum of each of these (L-)topologies is asymmetric. On the other hand, for each (complete) DeMorgan algebra L, $\mathbb{R}(L)$ with the canonical topology $\tau(L)$ is symmetric w.r.t. the first specialization order; and the "alternative" fuzzy real line $\mathbb{R}^*(L)$—the L-spectrum of the standard topology on $\mathbb{R}$—is symmetric in the same sense if L is a complete Boolean algebra. Under appropriate conditions, the well-known G_χ, M_L, ω_L, ι_L functors produce and/or reflect both asymmetric and symmetric L-topological spaces.

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