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by

Keiji Tagami

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Web:	http://topology.auburn.edu/tp/
Mail:	Topology Proceedings Department of Mathematics & Statistics Auburn University, Alabama 36849, USA
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KEIJI TAGAMI

Abstract. It is known that the maximal homological degree of the Khovanov homology of a knot gives a lower bound of the minimal positive crossing number of the knot. In this paper, we show that the maximal homological degree of the Khovanov homology of a cabling of a knot also gives a lower bound of the minimal positive crossing number of the knot.

1. Introduction

In [4], for each oriented link L, Mikhail Khovanov defined a graded chain complex whose graded Euler characteristic is equal to the Jones polynomial of L. Its homology is a link invariant and called the Khovanov homology. Throughout this paper, we denote the homological degree i term of the rational Khovanov homology of a link L by $\mathrm{KH}^i(L)$.

Let L be an oriented link. By $i_{\max}(L)$, we denote the maximal homological degree of the Khovanov homology of L, and by $c_+(L)$, we denote the minimal number of the positive crossings of diagrams of L. Note that $i_{\max}(L)$ is not negative. In fact, any link has nonzero Khovanov homology in degree zero because the Lee homology [6] is not trivial in homological degree zero. Then it is known that $i_{\max}(L)$ gives a lower bound of $c_+(L)$ (Proposition 2.1). From this fact, it seems that the Khovanov homology estimates the positivity of links.

Marko Stošić [9] showed that $i_{\max}(T_{2k,2kt})$ is $2k^2t$, where $T_{p,q}$ is the positive (p,q)-torus link. By using the same method as Stošić, the author [10] proved that $i_{\max}(T_{2k+1,(2k+1)t})$ is $2k(k+1)t$. The author also computed the maximal degree for a cabling of any knot.

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