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UNIFORMITY OF UNIFORM CONVERGENCE ON THE FAMILY OF SETS

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Abstract. We prove that for every Tychonoff space X and any uniform cub-space $(Y,\mathcal{U})$, the topology on $C(X,Y)$, induced by the uniformity $\hat{\mathcal{U}}|\lambda$ of uniform convergence on the saturation family λ, coincides with the set-open topology on $C(X,Y)$. For every Y^2-compact space X and any uniform space $(Y,\mathcal{U})$, the topology on $C(X,Y)$, induced by the uniformity $\hat{\mathcal{U}}$ of uniform convergence, coincides with the Y-compact-open topology on $C(X,Y)$ and depends only on the topology induced on Y by the uniformity $\mathcal{U}$. In particular, for every pseudocompact space X and any metrizable topological vector space Y with uniform $\mathcal{U}$, the topology on $C(X,Y)$, induced by the uniformity $\hat{\mathcal{U}}$ of uniform convergence, coincides with the C-compact-open topology on $C(X,Y)$ (with the compact-open topology on $C(\nu X,Y)$ where νX is the Hewitt real-compactification of X) and depends only on the topology induced on Y by the uniformity $\mathcal{U}$. It is also shown that in the class of closed-homogeneous complete uniform spaces Y, a necessary condition for coincidence of topologies is Y-compactness of the elements of the family λ.

1. Introduction

Let X be a Hausdorff space and let $(Y,\mathcal{U})$ be a uniform space. We shall denote by $C(X,Y)$ the set of all continuous mappings of the space X to the space Y, where Y is equipped with the topology induced by $\mathcal{U}$. For every

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