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SUBGROUPS GENERATED BY TWO DEHN TWISTS ON A NONORIENTABLE SURFACE

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Abstract. Let a and b be two simple closed curves on an orientable surface S such that their geometric intersection number is greater than 1. The group generated by corresponding Dehn twists t_a and t_b is known to be isomorphic to the free group of rank 2. In this paper we extend this result to the case of a nonorientable surface.

1. Introduction

Let N be a smooth, nonorientable, compact surface. We will mainly focus on the local properties of N; hence, we allow N to have some boundary components and/or punctures. Let $\mathcal{H}(N)$ be the group of all diffeomorphisms $h\colon N \to N$ such that h is the identity on each boundary component and h fixes the set of punctures (setwise). By $\mathcal{M}(N)$ we denote the quotient group of $\mathcal{H}(N)$ by the subgroup that comprises the maps isotopic to the identity with an isotopy which fixes the boundary pointwise. $\mathcal{M}(N)$ is known as the *mapping class group* of N. The mapping class group $\mathcal{M}(S)$ of an orientable surface S is defined analogously, but we consider only orientation preserving maps. Usually, we will use the same letter to denote a map and its isotopy class.

Important elements of the mapping class group $\mathcal{M}(S)$ are Dehn twists. Dehn twists generate $\mathcal{M}(S)$; thus, obtaining a good understanding of

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