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AN INTEGRAL WEIGHT REALIZATION THEOREM FOR SUBSET CURRENTS ON FREE GROUPS

ILYA KAPOVICH

Abstract. We prove that if $N \geq 2$ and $\alpha : F_N \to \pi_1(\Gamma)$ is a marking on F_N, then, for any integer $r \geq 2$ and any F_N-invariant collection of non-negative integral "weights" associated to all subtrees K of $\widetilde{\Gamma}$ of radius $\leq r$ satisfying some natural "switch" conditions, there exists a finite cyclically reduced folded Γ-graph Δ realizing these weights as numbers of "occurrences" of K in Δ. As an application, we give a new, direct, and explicit proof of one of the main results of our paper with Tatiana Nagnibeda (*Subset currents on free groups*, Geom. Dedicata **166** (2013), 307–348) stating that, for any $N \geq 2$, the set $\mathcal{S}\mathrm{Curr}_r(F_N)$ of all rational subset currents is dense in the space $\mathcal{S}\mathrm{Curr}(F_N)$ of subset currents on F_N. (The proof given in the above-cited paper was indirect and omitted significant details. The proof given here is complete and, we hope, more accessible to the $Out(F_N)$ community.)

We also answer one of the questions (Problem 10.11) posed in the above-mentioned paper. Thus, we prove that if a nonzero $\mu \in \mathcal{S}\mathrm{Curr}(F_N)$ has all weights with respect to some marking being integers, then μ is the sum of finitely many "counting" currents corresponding to nontrivial finitely generated subgroups of F_N.

1. Introduction

The main purpose of this paper is to give a proof of Theorem B below (originally established in [24] via an indirect argument) which is self-contained, direct, explicit, and can be relatively easily understood by the

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