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by

Heikki Junnila

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Web:	http://topology.auburn.edu/tp/
Mail:	Topology Proceedings Department of Mathematics & Statistics Auburn University, Alabama 36849, USA
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ON PARACOMPACT REMAINDERS

HEIKKI JUNNILA

Abstract. We study Tihonov spaces X which are *paracompact at infinity* (i.e., $\beta X \setminus X$ is paracompact). We characterize paracompactness at infinity of a nowhere locally compact Tihonov space, and we give several examples relating to paracompactness at infinity. We also consider strong paracompactness at infinity. We construct a space which is strongly paracompact at infinity but which also has a non-strongly paracompact remainder. We use this space to solve a problem on "paracompactly placed" sets posed by V.I. Ponomarev in 1962.

1. Introduction and notation

A *space* in this paper means a Tihonov space.

The *remainder* of a space X in a compactification K of X is the subspace $K \setminus X$ of K. We say that a space Z is a *remainder* of X provided that Z is the remainder of X in some compactification of X. The *Čech-Stone remainder* of X is the remainder of X in the Čech-Stone compactification βX of X; this remainder of X is denoted by X^*.

According to terminology introduced by Henriksen and Isbell in [16], a space X has *property P at infinity* if X^* has property P. The paper [16] contains the following characterization of Lindelöfness at infinity: X is Lindelöf at infinity if, and only if, every compact set $K \subset X$ is contained in a compact set $C \subset X$ such that C has a countable outer neighborhood base in X. As a consequence, every metrizable space is Lindelöf at infinity.

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