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A NIEMYTZKY-TYCHONOFF THEOREM FOR ALL TOPOLOGICAL SPACES

ITTAY WEISS

Abstract. The classical Niemytzky-Tychonoff theorem characterises compactness of a metrisable topological space by means of the completeness of all of the metrics inducing the topology. Motivated by results of Kopperman and Flagg to the effect that every topological space is metrisable, as long as metrisability is suitably modified to allow the metric to take values more general than real numbers, we show that the Niemytzky-Tychonoff theorem remains true under this broader notion of metrisability, thus obtaining a metric characterisation of compactness valid for all topological spaces.

1. Introduction

For a topological space (X, τ), we say that a metric $d\colon X \times X \to [0, \infty]$ is *compatible* if its induced open ball topology is τ.

Theorem 1.1 (Niemytzki-Tychonoff, 1928, [9])**.** *A metrisable topological space is compact if, and only if, it is complete in every compatible metric.*

It was popularised in [6] that if the metric function is allowed to take values in structures more general than the non-negative reals, and if the metric axioms are slightly relaxed, then every topological space is metrisable. In the literature, this result can be obtained in (at least) two ways, depending on the axioms defining the codomain of the metric function. Consequently, it is natural to contemplate the validity of the classical Niemytzki-Tychonoff theorem under this broader notion of metrisability.

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