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ULTRA STRONG S-SPACES

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Abstract. A strong S-space is an S-space X such that X^n is HS for all finite n. We consider replacing the "n" here with something infinite.

1. Introduction

All topological spaces considered in this paper are T_2 (Hausdorff).

A space X is *hereditarily separable* (*HS*) iff all subspaces of X are separable, and *hereditarily Lindelöf* (*HL*) iff all subspaces of X are Lindelöf. Also, X is *strongly* HS/HL iff X^n is HS/HL for all $n \in \omega$. Then, X is an *S-space* iff X is T_3 and HS but not HL, and X is a *strong S-space* iff in addition X is strongly HS.

S-spaces are consistent with $\mathsf{MA}(\aleph_1)$ [13], but are refuted by PFA [14]. On the other hand, strong S-spaces are refuted by $\mathsf{MA}(\aleph_1)$ [8], but exist under CH [11, 3]. For more background, see [12].

In this paper, we use $\diamondsuit$ to prove the existence of *ultra strong* S-spaces, satisfying a natural strengthening of strongly HS:

Definition 1.1. For topological spaces Q and X, let X^Q denote the space $C(Q, X)$ of continuous functions with the compact-open topology. Call X an *ultra strong S-space* iff X is an S-space and in addition X^Q is HS for all second countable compact Q.

Equivalently, an S-space X is ultra strong if X^Q is HS, where Q is the Cantor set; see Proposition 5.15.

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