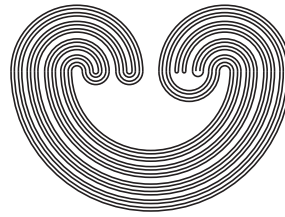


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## DEGREES OF MAPS BETWEEN ISOTROPIC GRASSMANN MANIFOLDS

by

SAMIK BASU AND SWAGATA SARKAR

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## DEGREES OF MAPS BETWEEN ISOTROPIC GRASSMANN MANIFOLDS

SAMIK BASU AND SWAGATA SARKAR

ABSTRACT. Let  $\tilde{I}_{2n,k}$  denote the space of  $k$ -dimensional, oriented isotropic subspaces of  $\mathbb{R}^{2n}$ , called the oriented isotropic Grassmannian. Let  $f: \tilde{I}_{2n,k} \rightarrow \tilde{I}_{2m,l}$  be a map between two oriented isotropic Grassmannians of the same dimension, where  $k, l \geq 2$ . We show that either  $(n, k) = (m, l)$  or  $\deg f = 0$ . Let  $\mathbb{R}\tilde{G}_{m,l}$  denote the oriented real Grassmann manifold. For  $k, l \geq 2$  and  $\dim \tilde{I}_{2n,k} = \dim \mathbb{R}\tilde{G}_{m,l}$ , we also show that the degree of maps  $g: \mathbb{R}\tilde{G}_{m,l} \rightarrow \tilde{I}_{2n,k}$  and  $h: \tilde{I}_{2n,k} \rightarrow \mathbb{R}\tilde{G}_{m,l}$  must be zero.

### 1. INTRODUCTION

It has been proved in [5] that maps between two different oriented real Grassmann manifolds of the same dimension cannot have non-zero degree, provided the target space is not a sphere. A similar result is obtained for complex Grassmann manifolds in [4], when the map is a morphism of projective varieties. For arbitrary maps, this result has been verified for the complex Grassmann manifolds for many cases in [5] and [6].

In this paper we consider the analogous question for the space  $\tilde{I}_{2n,k}$  of oriented  $k$ -dimensional isotropic subspaces of a symplectic vector space of dimension  $2n$ . The oriented isotropic Grassmannian was considered in [3] and its cohomology was computed with real coefficients. Their method involves identifying  $\tilde{I}_{2n,k}$  as a homogeneous space  $\tilde{I}_{2n,k} \simeq U(n)/(SO(k) \times U(n-k))$ . One may similarly consider  $I_{2n,k}$ , the isotropic Grassmannian of  $k$ -dimensional isotropic subspaces of a symplectic  $2n$  dimensional vector

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