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by

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DEGREES OF MAPS BETWEEN ISOTROPIC GRASSMANN MANIFOLDS

SAMIK BASU AND SWAGATA SARKAR

Abstract. Let $\widetilde{I}_{2n,k}$ denote the space of k-dimensional, oriented isotropic subspaces of $\mathbb{R}^{2n}$, called the oriented isotropic Grassmannian. Let $f\colon \widetilde{I}_{2n,k} \to \widetilde{I}_{2m,l}$ be a map between two oriented isotropic Grassmannians of the same dimension, where $k, l \geq 2$. We show that either $(n,k) = (m,l)$ or $\deg f = 0$. Let $\mathbb{R}\widetilde{G}_{m,l}$ denote the oriented real Grassmann manifold. For $k, l \geq 2$ and $\dim \widetilde{I}_{2n,k} = \dim \mathbb{R}\widetilde{G}_{m,l}$, we also show that the degree of maps $g\colon \mathbb{R}\widetilde{G}_{m,l} \to \widetilde{I}_{2n,k}$ and $h\colon \widetilde{I}_{2n,k} \to \mathbb{R}\widetilde{G}_{m,l}$ must be zero.

1. Introduction

It has been proved in [5] that maps between two different oriented real Grassmann manifolds of the same dimension cannot have non-zero degree, provided the target space is not a sphere. A similar result is obtained for complex Grassmann manifolds in [4], when the map is a morphism of projective varieties. For arbitrary maps, this result has been verified for the complex Grassmann manifolds for many cases in [5] and [6].

In this paper we consider the analogous question for the space $\widetilde{I}_{2n,k}$ of oriented k-dimensional isotropic subspaces of a symplectic vector space of dimension $2n$. The oriented isotropic Grassmannian was considered in [3] and its cohomology was computed with real coefficients. Their method involves identifying $\widetilde{I}_{2n,k}$ as a homogeneous space $\widetilde{I}_{2n,k} \simeq U(n)/(SO(k) \times U(n-k))$. One may similarly consider $I_{2n,k}$, the isotropic Grassmannian of k-dimensional isotropic subspaces of a symplectic $2n$ dimensional vector

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