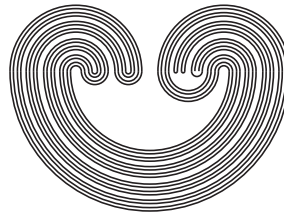


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PERSPECTIVES ON KUPERBERG FLOWS

by

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PERSPECTIVES ON KUPERBERG FLOWS

STEVEN HURDER AND ANA RECHTMAN

Dedicated to Professor Krystyna Kuperberg on the occasion of her 70th birthday

ABSTRACT. The “Seifert Conjecture” stated, “Every non-singular vector field on the 3-sphere \mathbb{S}^3 has a periodic orbit”. In a celebrated work, Krystyna Kuperberg gave a construction of a smooth aperiodic vector field on a plug, which is then used to construct counter-examples to the Seifert Conjecture for smooth flows on the 3-sphere, and on compact 3-manifolds in general. The dynamics of the flows in these plugs have been extensively studied, with more precise results known in special “generic” cases of the construction. Moreover, the dynamical properties of smooth perturbations of Kuperberg’s construction have been considered. In this work, we recall some of the results obtained to date for the Kuperberg flows and their perturbations. Then the main point of this work is to focus attention on how the known results for Kuperberg flows depend on the assumptions imposed on the flows, and to discuss some of the many interesting questions and problems that remain open about their dynamical and ergodic properties.

1. INTRODUCTION

In his 1950 work [59], Seifert introduced an invariant for deformations of non-singular flows with a closed orbit on a 3-manifold, which he used to show that every sufficiently small deformation of the Hopf flow on the 3-sphere \mathbb{S}^3 must have a closed orbit. He also remarked in Section 5 of this work:

It is unknown if every continuous (non-singular) vector field on the three-dimensional sphere contains a closed integral curve.

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