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## PRESERVATION OF A NEIGHBORHOOD BASE OF A SET BY CCC FORCINGS

by

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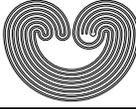
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## PRESERVATION OF A NEIGHBORHOOD BASE OF A SET BY CCC FORCINGS

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**ABSTRACT.** Let  $\langle X, \tau \rangle$  be a topological space and let  $A$  be a subset of  $X$ . We investigate under what circumstances a neighborhood base of  $A$  remains a neighborhood base of  $A$  in countable chain condition (ccc) forcing extensions.

We prove that if  $\langle X, \tau \rangle$  is a metrizable space and  $A \subseteq X$ , then the following are equivalent (Corollary 2.9):

- (1) Every forcing preserves a neighborhood base of  $A$ .
- (2) Every ccc forcing preserves a neighborhood base of  $A$ .
- (3) If  $B$  is the boundary of  $A$ , then  $B \cap A$  is scattered and compact.

### 1. INTRODUCTION

Let  $\mathbf{V}$  be a ground model and let  $\mathbb{P}$  be a forcing.  $\mathbf{V}^{\mathbb{P}}$  denotes the forcing extension of  $\mathbf{V}$  by the forcing  $\mathbb{P}$ . For a topological space  $\langle X, \tau \rangle$  in  $\mathbf{V}$ , we define a topological space  $\langle X, \tau^{\mathbb{P}} \rangle$  in  $\mathbf{V}^{\mathbb{P}}$  such that

$\tau^{\mathbb{P}}$  is the topology on  $X$  generated by  $\tau$  in  $\mathbf{V}^{\mathbb{P}}$ .

Observe that, in general,  $\tau \subsetneq \tau^{\mathbb{P}}$  (because the forcing  $\mathbb{P}$  introduces new open sets) and that  $\tau$  serves as a base for  $\tau^{\mathbb{P}}$  by definition.

We say that a topological property  $\varphi$  is preserved by forcing if, whenever a space  $\langle X, \tau \rangle$  satisfies  $\varphi$ ,  $\langle X, \tau^{\mathbb{P}} \rangle$  satisfies  $\varphi$  for any forcing  $\mathbb{P}$ . Topological properties such as Hausdorffness, regularity, and complete regularity are preserved by forcing ([2, Lemma 22]), but normality may not be preserved by forcing ([10, Theorem 1.8]). Renata Grunberg, Lúcia

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