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TOPOLOGY PROCEEDINGS

Volume 52, 2018

Pages 101–112

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Electronically published on September 1, 2017

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Topology Proceedings

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E-mail:	topolog@auburn.edu
ISSN:	(Online) 2331-1290, (Print) 0146-4124

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TOPOLOGY PROCEEDINGS
Volume 52 (2018)
Pages 101-112

E-Published on September 1, 2017

REOPENING SEVERAL QUESTIONS CONCERNING $\mathcal{P}$-CLOSED SPACES

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Abstract. Examples are given to point out gaps or errors in certain recently published assertions, proofs, and answers to questions concerning $\mathcal{P}$-closed spaces. In particular, examples are given to show that it is still unknown if a Urysohn (regular) space in which every closed subset is Urysohn-closed (regular-closed) must be compact.

1. Introduction and Terminology

All hypothesized spaces are Hausdorff. A space in which any two distinct points have disjoint closed neighborhoods is called a *Urysohn space*. We recall that for a topological property $\mathcal{P}$, a $\mathcal{P}$-space which is a closed subspace of any $\mathcal{P}$-space in which it can be embedded is called $\mathcal{P}$-closed. In [13] and [15] proofs were given that every space in which every closed subset is Hausdorff-closed is compact. Other researchers later raised analogous questions by asking if every space in which every closed subset is Urysohn-closed must be compact [2] and if every space in which every closed subset is regular-closed must be compact [1].

In the recent articles, James E. Joseph and Bhamini M. P. Nayar [12] and Terrence A. Edwards, et al. [6] present proofs that for $\mathcal{P}$ = Hausdorff, Urysohn, or regular, a $\mathcal{P}$-space in which every closed subset is $\mathcal{P}$-closed must be compact. In this note we point out, however, that certain spaces and filter bases provide counterexamples to some of their assertions and proofs, and, consequently, the latter two questions above are not settled in their articles. We also review several other assertions, proofs, and

2010 *Mathematics Subject Classification.* Primary 54D25, 54D30; Secondary 54B10.

Key words and phrases. H-closed, regular-closed, Urysohn-closed.
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