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ON ISOTOPY OF SELF-HOMEOMORPHISMS OF QUADRATIC INVERSE LIMIT SPACES

H. BRUIN AND S. ŠTIMAC

Abstract. We prove that every self-homeomorphism on the inverse limit space of a quadratic map is isotopic to some power of the shift map.

1. Introduction

The two most prominent families of unimodal maps are the family of quadratic maps Q_a, $a \in [1,4]$, and the family of tent maps T_s, $s \in [1,2]$. The inverse limit spaces of quadratic and tent maps share a lot of common properties. For example, if f is a map from one of these families, then 0 is a fixed point of f; the point $\bar{0} := (\dots,0,0,0)$ is contained in $\varprojlim([0,1],f)$ and is an end-point. The arc-component C of $\varprojlim([0,1],f)$ which contains $\bar{0}$ is a ray converging to, but (provided $a<4$ and $s<2$) disjoint from the inverse limit of the core $\varprojlim([c_2,c_1],f)$, and $\varprojlim([0,1],f) = C \cup \varprojlim([c_2,c_1],f)$, where the critical or turning point is denoted as c and $c_k := f^k(c)$. If c is periodic with (prime) period N, then $\varprojlim([c_2,c_1],f)$ contains N end-points.

The relationships between quadratic and tent maps and between their inverse limits are mostly well understood. Each quadratic map Q_a with positive topological entropy is semi-conjugate to a tent map T_s with $\log s = h_{top}(Q_a)$, and this semi-conjugacy collapses (pre)periodic intervals to points [6]. If a quadratic map is not renormalizable and does not

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