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TOPOLOGY PROCEEDINGS

Volume 52, 2018

Pages 219–233

http://topology.nipissingu.ca/tp/

Finite Graphs Have Unique n-Fold Pseudo-Hyperspace Suspension

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Electronically published on January 22, 2018

Topology Proceedings

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E-mail:	topolog@auburn.edu
ISSN:	(Online) 2331-1290, (Print) 0146-4124

http://topology.auburn.edu/tp/
http://topology.nipissingu.ca/tp/

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Pages 219-233

E-Published on January 22, 2018

FINITE GRAPHS HAVE UNIQUE n-FOLD PSEUDO-HYPERSPACE SUSPENSION

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Abstract. Let X be a metric continuum. Let n be a positive integer, let $C_n(X)$ be the space of all nonempty closed subsets of X with at most n components, and let $F_1(X)$ be the space of singletons of X. The *n-fold pseudo-hyperspace suspension* of X is the quotient space $C_n(X)/F_1(X)$ and it is denoted by $PHS_n(X)$. In this paper we prove that if X is a finite graph and Y is a continuum such that $PHS_n(X)$ is homeomorphic to $PHS_n(Y)$, then X is homeomorphic to Y. This answers a question by Juan C. Macías.

1. Introduction

A *continuum* is a compact connected metric space. For a continuum X, consider the following set:

$2^X = \{A \subset X : A \text{ is a nonempty closed subset of } X\}$.

Let n be a positive integer; the *n-fold hyperspace* of X, denoted by $C_n(X)$, is the set:

$\{A \in 2^X : A \text{ has at most } n \text{ components}\}$;

the *n-fold symmetric product* of X, denoted by $F_n(X)$, is the set:

$\{A \subset X : A \text{ has at most } n \text{ points}\}$.

These sets are topologized with the Hausdorff metric which is defined as

$$\mathcal{H}(A,B) = \inf\{\varepsilon > 0 : A \subset \mathcal{V}_\varepsilon(B) \text{ and } B \subset \mathcal{V}_\varepsilon(A)\},$$

where $\mathcal{V}_\varepsilon(A) = \{x \in X : d(x,A) < \varepsilon\}$.

Sam B. Nadler, Jr., [21] introduced the hyperspace suspension of a continuum, $HS(X)$, as the quotient space $C_1(X)/F_1(X)$. Later Sergio

2010 *Mathematics Subject Classification.* 54B20.

Key words and phrases. Continuum, n-fold hyperspace, n-fold hyperspace suspension, n-fold pseudo-hyperspace suspension, unique hyperspace.

The research in this paper is part of the author's Ph.D. Dissertation at Universidad Nacional Autónoma de México, under the supervision of Dr. Sergio Macías.