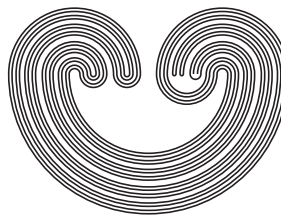


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TOPOLOGY PROCEEDINGS



Volume 52, 2018

Pages 235–244

<http://topology.nipissingu.ca/tp/>

ON ORDERED TOPOLOGICAL VECTOR SPACES WITH POSITIVE INTERIOR POINTS

by

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Electronically published on February 5, 2018

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E-mail: topolog@auburn.edu

ISSN: (Online) 2331-1290, (Print) 0146-4124

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ON ORDERED TOPOLOGICAL VECTOR SPACES WITH POSITIVE INTERIOR POINTS

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ABSTRACT. Answering a question indicated by Er-Guang Yang in (*Partial answers to some questions on maps to ordered topological vector spaces*, Topology Proc. **50** (2017), 311–317), we show that, for an ordered topological vector space Y with positive interior points, if each non-zero positive element is an order unit, then Y is isomorphic to the real line. We also provide a technique which reduces some vector-valued results to the original real-valued ones by using some Minkowski functionals.

1. INTRODUCTION

Throughout this paper, let \mathbb{R} be the set of all real numbers, and \mathbb{N} the set of all natural numbers.

Let us recall some terminology from [1] and [5]. A partially ordered real vector space (Y, \leq) is said to be an *ordered vector space* if the following conditions are satisfied:

- (i) $x \leq y$ implies $x + z \leq y + z$ for all $x, y, z \in Y$,
- (ii) $x \leq y$ implies $rx \leq ry$ for all $x, y \in Y$ and all $r \in \mathbb{R}$ with $r \geq 0$.

Let (Y, \leq) be an ordered vector space. Then, $y \in Y$ is *positive* if $\mathbf{0} \leq y$, and the set $\{y \in Y : \mathbf{0} \leq y\}$, called the *positive cone* of Y , is denoted by Y^+ . For $y_1, y_2 \in Y$ with $y_1 \leq y_2$, the subspace $(y_1 + Y^+) \cap (y_2 - Y^+)$ of Y , called an *order interval*, is denoted by $[y_1, y_2]$. A topological vector space Y is called an *ordered topological vector space (o.t.v.s.)* if Y is an ordered vector space such that the positive cone Y^+ is closed in Y . It is

2010 *Mathematics Subject Classification.* Primary 54C08, 54D20, 46A40.

Key words and phrases. ordered topological vector space, positive interior point, order unit, Minkowski functional, monotonically countably paracompact.

This work was supported by TCUE Special Grant-in-Aid (A) 2017.

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