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## ON SPACES WITH $\sigma$ -CLOSED-DISCRETE DENSE SETS

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## ON SPACES WITH $\sigma$ -CLOSED-DISCRETE DENSE SETS

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**ABSTRACT.** The main purpose of this paper is to study  $e$ -separable spaces, originally introduced by Georges Kurepa as  $K'_0$  spaces; we call a space  $X$   $e$ -separable if and only if  $X$  has a dense set which is the union of countably many closed discrete sets. We primarily focus on the behavior of  $e$ -separable spaces under products and the cardinal invariants that are naturally related to  $e$ -separable spaces. Our main results show that the statement “there is a product of at most  $\mathfrak{c}$  many  $e$ -separable spaces that fails to be  $e$ -separable” is overinsistent with the existence of a weakly compact cardinal.

### 1. INTRODUCTION

The goal of this paper is to study a natural generalization of separability. Let us call a space  $X$   $e$ -separable if and only if  $X$  has a dense set which is the union of countably many closed discrete sets. The definition is due to Georges Kurepa [18], who introduced this notion as property  $K'_0$  in his study of Souslin’s problem. Later,  $e$ -separable spaces appear in multiple papers related to the study of linearly ordered and GO-spaces [11], [25], [26], [30]. In particular, M. J. Faber [11] showed that  $e$ -separable GO-spaces are perfect; however, whether the converse implication is true is famously open: is there, in ZFC, a perfect GO-space (or even just a perfect  $T_3$  space) which is not  $e$ -separable? Let us refer the interested reader to a paper of Harold Bennett and David Lutzer [5] for more details and results on this topic.

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