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ON STIEFEL–WHITNEY CLASSES OF VECTOR BUNDLES OVER REAL STIEFEL MANIFOLDS

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Abstract. In this article, we show that there are at most two integers up to $2(n-k)$, which can occur as the degrees of nonzero Stiefel–Whitney classes of vector bundles over the Stiefel manifold $V_k(\mathbb{R}^n)$. In the case when $n > k(k+4)/4$, we show that if $w_{2^q}(\xi)$ is the first nonzero Stiefel–Whitney class of a vector bundle ξ over $V_k(\mathbb{R}^n)$, then $w_t(\xi)$ is zero if t is not a multiple of 2^q. In addition, we give relations among Stiefel–Whitney classes whose degrees are multiples of 2^q.

1. Introduction

The real Stiefel manifold $V_k(\mathbb{R}^n)$ is the set of all orthonormal k-frames in $\mathbb{R}^n$, and it can be identified with the homogeneous space $SO(n)/SO(n-k)$. The main aim of this article is to study Stiefel–Whitney classes of vector bundles over a real Stiefel manifold.

Recall that the degree of the first nonzero Stiefel–Whitney class of a vector bundle over a CW-complex X is a power of 2 (see, for example, [8, p. 94]). In the case when X is a d-dimensional sphere S^d, M. F. Atiyah and F. Hirzebruch [2, Theorem 1] show that d can occur as the degree of a nonzero Stiefel–Whitney class of a vector bundle over S^d if and only if $d = 1, 2, 4, 8$. The possible Stiefel–Whitney classes of vector bundles over Dold manifold and stunted real projective space are completely determined by R. E. Stong [11] and Ryuichi Tanaka [12], respectively. In this article, we shall

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