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A BANACH-STONE TYPE THEOREM FOR C^1-FUNCTION SPACES OVER THE CIRCLE

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Abstract. We prove and apply an elementary theorem in calculus to improve the Banach-Stone type theorem on C^1-function spaces over the unit circle on the complex plane proved in [8, Theorem 3.1].

1. Introduction

This is a continuation of the paper [8]. The Banach-Stone theorem states that every linear isometry on the space of continuous functions over a compact Hausdorff space (with the sup norm) is a weighted composition operator with a unimodular weight. Various extensions of the theorem have been studied by many authors (see monographs [3],[4]). Banach-Stone type theorems for the C^1-function spaces over $[0, 1]$ have been proved in [1], [6], [7], [11], [12], [13], [14] etc. Recently the author obtained a similar theorem for $C^1(\mathbb{T})$, the space of C^1-function space over the unit circle $\mathbb{T} = \{z \in \mathbb{C} \mid |z| = 1\}$ on the complex plane [8]. In [8, Theorem 3.1] it was shown that every linear isometry $T : C^1(\mathbb{T}) \to C^1(\mathbb{T})$ with respect to a suitable norm is a weighted composition operator or its variant if the isometry T satisfies an additional hypothesis:

$$(*)\ T(\mathrm{id}_{\mathbb{T}}) \text{ and } T(\overline{\mathrm{id}_{\mathbb{T}}}) \text{ are } C^3\text{-functions}$$

where $\mathrm{id}_{\mathbb{T}}$ and $\overline{\mathrm{id}}_{\mathbb{T}}$ denote the identity function on $\mathbb{T}$ and its complex conjugate respectively. The above technical hypothesis was assumed only to prove the following lemma.

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