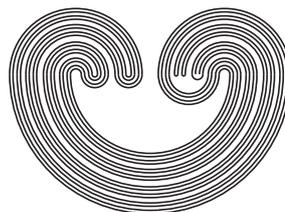


<http://topology.auburn.edu/tp/>

TOPOLOGY PROCEEDINGS



Volume 53, 2019

Pages 27–36

<http://topology.nipissingu.ca/tp/>

SPACES WITH NO INFINITE DISCRETE SUBSPACE

by

JEAN GOUBAULT-LARRECQ

Electronically published on March 18, 2018

This file contains only the first page of the paper. The full version of the paper is available to Topology Proceedings subscribers. See <http://topology.auburn.edu/tp/subscriptioninfo.html> for information.

Topology Proceedings

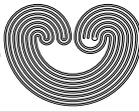
Web: <http://topology.auburn.edu/tp/>

Mail: Topology Proceedings
Department of Mathematics & Statistics
Auburn University, Alabama 36849, USA

E-mail: topolog@auburn.edu

ISSN: (Online) 2331-1290, (Print) 0146-4124

COPYRIGHT © by Topology Proceedings. All rights reserved.



SPACES WITH NO INFINITE DISCRETE SUBSPACE

JEAN GOUBAULT-LARRECQ

ABSTRACT. We show that the spaces with no infinite discrete subspace are exactly those in which every closed set is a finite union of irreducibles. Call them FAC spaces: this generalizes a theorem by Erdős and Tarski (1943), according to which a preordered set has no infinite antichain—the finite antichain, or FAC, property—if and only if all its downwards-closed subsets are finite unions of ideals. All Noetherian spaces are FAC spaces, and we show that sober FAC spaces have a simple order-theoretic description.

1. INTRODUCTION

A preorder is *FAC* (for: has the *finite antichain* property) if and only if all its antichains are finite. An antichain is a subset of pairwise incomparable elements. A well-known result in the theory of preorders states that a preorder is FAC if and only if all its downwards-closed subsets are finite unions of ideals—an ideal is a downwards-closed, directed subset. This was discovered many times [1, 9, 10, 3, 7], and is credited to Erdős and Tarski [2].

The purpose of this paper is to generalize that result to the case of topological spaces. An antichain will simply be a discrete subspace, namely a subspace whose topology is discrete. Downwards-closed subsets will be replaced by closed subsets, and ideals by irreducible closed subsets. We shall retrieve the above preorder-theoretic result by looking at spaces with the Alexandroff topology of the preorder. This is in line with related results, such as the fact that Noetherian spaces are a topological generalization of well-quasi-orders [4].

2010 *Mathematics Subject Classification.* Primary 54G99; Secondary 06A07, 06B30.

Key words and phrases. Discrete subspace, irreducible closed subset, finite antichain, ideal, sober space, Noetherian space.

The author was supported by grant ANR-17-CE40-0028 of the French National Research Agency ANR (project BRAVAS).

©2018 Topology Proceedings.

This file contains only the first page of the paper. The full version of the paper is available to Topology Proceedings subscribers. See <http://topology.auburn.edu/tp/subscriptioninfo.html> for information.