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TOPOLOGY PROCEEDINGS

Volume 53, 2019

Pages 47–56

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# The universality of three-dimensional subdivision rules

by

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Electronically published on May 4, 2018



**Topology Proceedings**

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| **E-mail:** | topolog@auburn.edu |
| **ISSN:** | (Online) 2331-1290, (Print) 0146-4124 |

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# THE UNIVERSALITY OF THREE-DIMENSIONAL SUBDIVISION RULES

BRIAN RUSHTON

Abstract. We characterize the history graph of a finite subdivision rule in terms of its combinatorics. We use this to show that each finite subdivision rule is combinatorially equivalent to a three-dimensional finite subdivision rule. This shows that high-dimensional recursive sequences of cell complexes (such as those used to construct higher-dimensional analogues of the Sierpinski cube) have the same adjacency patterns as 3-dimensional sequences, which are easier to visualize.

## 1. Introduction

Finite subdivision rules are a very general construct for creating recursively defined structures in all dimensions (such as the Sierpinski triangle, carpet, and cube). They consist of a set of topological spaces and maps used recursively to create more and more refined cell structures. In this paper, we show that a finite subdivision rule (in any dimension) is combinatorially equivalent to a three-dimensional finite subdivision rule (in the sense that there is map between the two recursive sequences of cell structures that preserves adjacency of cells).

The main difficulty in the paper is not in finding a sequence of 3-dimensional cell structures with the same adjacencies, but in finding such a sequence that is defined recursively, i.e. another finite subdivision rule.

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2010 *Mathematics Subject Classification.* 52C26, 05C63.

*Key words and phrases.* Subdivision rules, subdivision rule, cell complex, low-dimensional topology.