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HIGHER ORDER ELLIPTIC FUNCTIONS WITH CONNECTED JULIA SETS

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Abstract. Connectivity properties of elliptic functions are completely understood for only four families of order two elliptic functions. In this paper, we find families of elliptic functions of arbitrarily high order on square and triangular lattices for which the Julia set is connected.

There are four families of elliptic functions for which the connectivity of the Julia set is completely understood. Let $\wp_\Lambda$ denote the Weierstrass elliptic function on the lattice Λ. The Julia set of $\wp_\Lambda$ is always connected if Λ is a triangular [8] or square [3, 5] lattice. The Julia set of $1/\wp_\Lambda$ is either connected or Cantor if Λ is a triangular lattice [10] or a square lattice [13]. Connectivity properties of the Julia set of certain higher order elliptic functions on real rectangular lattices appear in [9, 11], but these results do not cover all of the functions in those families.

Both $\wp_\Lambda$ and $1/\wp_\Lambda$ have order two. The proofs of the connectivity results rely on using properties of the special triangular and square lattice shapes to investigate the locations of the critical values. Symmetry properties of $\wp_\Lambda$ on a triangular or square lattices force any Fatou component to contain at most one critical value. These families contain functions for which the Fatou set is nonempty as well as functions for which the Julia set is the entire sphere. On the other hand, the functions $1/\wp_\Lambda$ on a triangular or square lattice always have a super attracting Fatou component at the origin. Again, symmetry properties of the function imply that either this Fatou component contains all of the critical values, resulting in a Cantor Julia set, or every Fatou component contains at most one critical value, resulting in a connected Julia set.

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